

$$\begin{aligned}
\sum_{\mathbb{J}} -\mathbb{J}^{\dagger}\mathbb{F}\mathbb{J}\mathbf{e} &\stackrel{\text{Pois}}{=} \sum_{\mathbb{I}} \int_{\mathbb{J}}^{d_{\mathbb{R}}} \mathbb{I}^{\mathbb{J}} - \mathbb{J}^{\dagger}\mathbb{F}\mathbb{J}\mathbf{e} = \sum_{\mathbb{I}} \int_{\mathbb{J}}^{d_{\mathbb{R}}} 2\pi i\mathbb{I}^{\mathbb{J}} - \mathbb{J}^{\dagger}\mathbb{F}\mathbb{J}\mathbf{e} \\
&\int_{\mathbb{J}}^{d_{\mathbb{R}}} 2i\mathbb{J}^{\mathbb{J}} - \mathbb{J}^{\dagger}T\mathbb{J}\mathbf{e} = \frac{\pi^{d/2}}{\sqrt{T}^{1/2}} -\mathbb{J}^{\dagger}\mathcal{T}\mathbb{J}\mathbf{e} \\
&\int_{\mathbb{J}}^{d_{\mathbb{R}}} 2\mathbb{J}^{\mathbb{J}} - \mathbb{J}^{\dagger}T\mathbb{J}\mathbf{e} = \frac{\pi^{d/2}}{\sqrt{T}^{1/2}} \mathbb{J}^{\dagger}\mathcal{T}\mathbb{J}\mathbf{e} \\
\sum_{\mathbb{J}} -\mathbb{J}^{\dagger}\mathbb{F}\mathbb{J}\mathbf{e} &= \sum_{\mathbb{I}} \int_{\mathbb{J}}^{d_{\mathbb{R}}} 2\pi i\mathbb{I}^{\mathbb{J}} - \mathbb{J}^{\dagger}\mathbb{F}\mathbb{J}\mathbf{e} = \frac{\pi^{d/2}}{\sqrt{\mathbb{J}}^{1/2}} \sum_{\mathbb{I}} -\pi\mathbb{I}^{\mathbb{J}}\overline{\pi^{\dagger}}\mathbf{e} = \frac{\pi^{d/2}}{\sqrt{\mathbb{J}}^{1/2}} \sum_{\mathbb{I}} -\mathbb{I}^{\mathbb{J}}\overline{\mathbb{F}}\mathbb{I}^{\dagger}\mathbf{e} \\
\sum_{\mathbb{J}} -\pi\mathbb{J}^{\dagger}\mathbb{F}\mathbb{J}\mathbf{e} &= \frac{\pi^{d/2}}{\sqrt{\pi\mathbb{J}}^{1/2}} \sum_{\mathbb{I}} -\mathbb{I}^{\mathbb{J}}(\pi^{\dagger}\mathbb{F})\mathbb{I}^{\dagger}\mathbf{e} = \sqrt{\mathbb{J}}^{-1/2} \sum_{\mathbb{I}} -\pi\mathbb{I}^{\mathbb{J}}\overline{\mathbb{F}}\mathbb{I}^{\dagger}\mathbf{e}
\end{aligned}$$