$$\mathcal{L}_X^h = h^{\alpha\beta}\,\partial_\alpha X \cdot \partial_\beta X$$

$$\partial^{\alpha\beta}\sqrt{-\det h}\,\mathcal{L}_X^h = \sqrt{-\det h}\left(h^{\alpha\gamma}\partial_\gamma X \cdot h^{\beta\delta}\partial_\delta X - \frac{1}{2}h^{\alpha\beta}\mathcal{L}_X^h\right)$$

$$\partial^{\alpha\beta}\det h = h^{\alpha\beta}\det h$$

$$\partial^{\alpha\beta}\,()^{\gamma\delta} = -h^{\alpha\gamma}\,h^{\beta\delta}$$

$$\begin{aligned}\partial^{\alpha\beta}\sqrt{-\det h}\,h^{\gamma\delta}\,\partial_\gamma X \cdot \partial_\delta X &= \frac{1}{2\sqrt{-\det h}}\partial^{\alpha\beta}\left(-\det h\right)h^{\gamma\delta}\,\partial_\gamma X \cdot \partial_\delta X + \sqrt{-\det h}\,\partial^{\alpha\beta}\,()^{\gamma\delta}\,\partial_\gamma X \cdot \partial_\delta X \\ &= \frac{1}{2\sqrt{-\det h}}\left(-\det h\right)h^{\alpha\beta}\,h^{\gamma\delta}\,\partial_\gamma X \cdot \partial_\delta X - \sqrt{-\det h}\,h^{\alpha\gamma}\,h^{\beta\delta}\,\partial_\gamma X \cdot \partial_\delta X \\ &= \sqrt{-\det h}\left(\frac{1}{2}h^{\alpha\beta}h^{\gamma\delta}\partial_\gamma X \cdot \partial_\delta X - h^{\alpha\gamma}h^{\beta\delta}\partial_\gamma X \cdot \partial_\delta X\right)\end{aligned}$$