$$\begin{bmatrix}\varphi\\ =\end{bmatrix} \frac{\overbrace{\partial_t\varphi}^{2} - \overbrace{\partial_x\varphi}^{2}}{2} + \varphi^2 - \frac{1}{2}\varphi^4$$

$$^{t:x}\varphi = {}^{t/m:x/m}\phi\,\frac{\sqrt{f}}{m}$$

$$\partial_t\varphi = \partial_t\phi\frac{\sqrt{f}}{m^2}$$

$$\partial_x\varphi = \partial_x\phi\frac{\sqrt{f}}{m^2}$$

$$\frac{\overbrace{\partial_t\varphi}^{2} - \overbrace{\partial_x\varphi}^{2}}{2} + \varphi^2 - \frac{1}{2}\varphi^4 = \frac{f}{m^4}\frac{\overbrace{\partial_t\phi}^{2} - \overbrace{\partial_x\phi}^{2}}{2} + \frac{f}{m^2}\phi^2 - \frac{f^2}{2m^4}\phi^4 = \frac{f}{m^4}\left(\frac{\overbrace{\partial_t\phi}^{2} - \overbrace{\partial_x\phi}^{2}}{2} + \frac{m^2}{2}\text{ mass }\phi^2 - \frac{f/2}{2}\text{ coupling }\phi^4\right)$$

$$\mathbb{R}^{1:1} \xrightarrow{\varphi} \mathbb{R} \text{ real scalar field}$$

$$\text{motion } \partial_t^2\varphi - \partial_x^2\varphi = \partial_\mu\,\partial^\mu\,\mathcal{L}_\varphi = \partial\,\mathcal{L}_\varphi = 2\varphi - 2\varphi^3$$

$$\text{constant solutions } \varphi = \varphi^3 \Longrightarrow \varphi = \begin{cases}0\\ \pm 1\end{cases}$$

$$\text{energy-density } {}_0\mathcal{L}^0_\varphi = \underbrace{\partial_0\varphi}\,\underbrace{\delta^0\mathcal{L}_\varphi} - \mathcal{L}_\varphi = \underbrace{\partial_0^2\varphi} - \mathcal{L}_\varphi = \frac{\overbrace{\partial_t\varphi}^{2} + \overbrace{\partial_x\varphi}^{2}}{2} + \frac{\varphi^4}{2} - \varphi^2$$

$$\text{Vac 0-sphere } \overset{\pm 1}{=} \mathbb{S}_0$$

$$V(\varphi) = \frac{\varphi^4}{2} - \varphi^2 \Longrightarrow \underline{V}(\varphi) = 2\varphi^3 - 2\varphi \Longrightarrow \underline{\underline{V}}(\varphi) = 6\varphi^2 - 2$$

$$\begin{cases}\underline{\underline{V}}(0) = -2 < 0 & \Longrightarrow \text{ max=anti-vac}\\ \underline{\underline{V}}(\pm 1) = 4 > 0 & \Longrightarrow \text{ min=vac}\end{cases}$$

$$\text{vacuum density } {}_0\mathcal{L}^0{}_{\text{vac}} = -\frac{1}{2}$$

$${}_0\mathcal{L}^0_\varphi - {}_0\mathcal{L}^0{}_{\text{vac}} = \frac{\overbrace{\partial_t\varphi}^{2} + \overbrace{\partial_x\varphi}^{2}}{2} + \frac{\varphi^4}{2} - \varphi^2 + \frac{1}{2} = \frac{\overbrace{\partial_t\varphi}^{2} + \overbrace{\partial_x\varphi}^{2}}{2} + \frac{\left(\varphi^2 - 1\right)^2}{2}$$

$$ {}_0L^{0}_{\varphi} - {}_0L^{0}_{\text{vac}} = \int_{\mathbb{R}}^{dx} \frac{\overbrace{\partial_t\varphi}^{2} + \overbrace{\partial_x\varphi}^{2}}{2} + \frac{\left(\varphi^2 - 1\right)^2}{2} $$

finite energy solutions

$$ \int_{\mathbb{R}}^{dx} \overbrace{\partial_t\varphi}^{2} + \overbrace{\partial_x\varphi}^{2} + \left(\varphi^2 - 1\right)^2 < \infty \Longrightarrow $$

$$ \int_{\mathbb{R}}^{dx} \overbrace{\partial_t\varphi}^{2} < \infty \Longrightarrow \partial_t \varphi \ {}_{x}\!\rightsquigarrow_{\infty}\ 0 $$

$$ \int_{\mathbb{R}}^{dx} \overbrace{\partial_x\varphi}^{2} < \infty \Longrightarrow \partial_x \varphi \ {}_{x}\!\rightsquigarrow_{\infty}\ 0 $$

$$ \int_{\mathbb{R}}^{dx} \left(\varphi^2 - 1\right)^2 < \infty \Longrightarrow {}^{x}\varphi^2 \ {}_{x}\!\rightsquigarrow_{\infty}\ 1 \Longrightarrow {}^{x}\varphi \ {}_{x}\!\rightsquigarrow_{\infty}\ {}_{=}\pm \ \text{vac} $$

static solutions

$$ \partial_t \varphi = 0 \Longrightarrow \partial_x^2 \varphi + 2\varphi - 2\varphi^3 = 0 \Longrightarrow \partial_x \underbrace{\overbrace{\partial_x\varphi}^{2} + 2\varphi^2 - \varphi^4} = 2\,\partial_x \varphi \underbrace{\partial_x^2\varphi + 2\varphi - 2\varphi^3} = 0 $$

$$ \Longrightarrow \overbrace{\partial_x\varphi}^{2} + 2\varphi^2 - \varphi^4 = -C \Longrightarrow \overbrace{\partial_x\varphi}^{2} = \varphi^4 - 2\varphi^2 + C \Longrightarrow \frac{d\varphi}{dx} = \sqrt{\varphi^4 - 2\varphi^2 + C} \Longrightarrow dx = \frac{d\varphi}{\sqrt{\varphi^4 - 2\varphi^2 + C}} $$

static finite energy solutions

$$ C = \overbrace{\partial_x\varphi}^{2} + 2\varphi^2 - \varphi^4 \ {}_{x}\!\rightsquigarrow_{\infty}\ 0 + 2 - 1 = 1 \Longrightarrow C = 1 \Longrightarrow \overbrace{\partial_x\varphi}^{2} = \varphi^4 - 2\varphi^2 + 1 = \left(1 - \varphi^2\right)^2 $$

$$ \Longrightarrow \frac{d\varphi}{dx} = \pm\left(1 - \varphi^2\right) \Longrightarrow dx = \pm\frac{d\varphi}{1 - \varphi^2} \Longrightarrow x - x_0 = \pm\int \frac{d\varphi}{1 - \varphi^2} = \pm\, {}^{\varphi}\overset{-1}{\tanh} \Longrightarrow {}^{x}\varphi = \pm\, {}^{x - x_0}\tanh $$