

$$\begin{aligned} & \overbrace{L_0^{-1}} V_1 \overbrace{L_0^{-1}} V_2 \overbrace{L_0^{-1}} \cdots \overbrace{L_0^{-1}} V_m \\ &= \underbrace{\sqrt{x_1}^{P^2} \sqrt{x_2}^{\widehat{P-k_1}^2} \sqrt{x_3}^{\widehat{P-k_1-k_2}^2} \cdots \sqrt{x_m}^{\widehat{P-k_1-k_2-\cdots-k_{m-1}}^2}}_{\mathbf{x} \underbrace{x_1^L V_1 x_2^L V_2 x_3^L \cdots x_m^L V_m}_{\mathbf{x}}} \end{aligned}$$

$$\begin{aligned} \text{LHS} &= x_1^L V_1 x_2^L V_2 x_3^L \cdots x_m^L V_m = \underbrace{\sqrt{x_1}^{P^2} \mathbf{x} x_1^L k_1 \mathbf{x} V_1}_{\mathbf{x}} \underbrace{\sqrt{x_2}^{P^2} \mathbf{x} x_2^L k_2 \mathbf{x} V_2}_{\mathbf{x}} \underbrace{\sqrt{x_3}^{P^2} \mathbf{x} x_3^L \cdots \sqrt{x_m}^{P^2} \mathbf{x} x_m^L k_m \mathbf{x} V_m}_{\mathbf{x}} \\ &= \underbrace{\sqrt{x_1}^{P^2} k_1 \sqrt{x_2}^{P^2} k_2 \sqrt{x_3}^{P^2} k_3 \cdots \sqrt{x_m}^{P^2} k_m}_{\mathbf{x}} \mathbf{x} \underbrace{x_1^L V_1 x_2^L V_2 x_3^L \cdots x_m^L V_m}_{\mathbf{x}} = \text{RHS} \end{aligned}$$

$$\begin{aligned} & \text{Tr } \overbrace{L_0^{-1}} V_1 \overbrace{L_0^{-1}} V_2 \overbrace{L_0^{-1}} \cdots \overbrace{L_0^{-1}} V_m \\ &= \text{Tr } \underbrace{\sqrt{x_1}^{P^2} \sqrt{x_2}^{\widehat{P-k_1}^2} \sqrt{x_3}^{\widehat{P-k_1-k_2}^2} \cdots \sqrt{x_m}^{\widehat{P-k_1-k_2-\cdots-k_{m-1}}^2}}_{\mathbf{x}} \text{Tr } \underbrace{x_1^L V_1 x_2^L V_2 x_3^L \cdots x_m^L V_m}_{\mathbf{x}} \\ & \sum_i^{1|m} \overbrace{P - k_1 - \cdots - k_{i-1}}^2 x_i \mathbf{x} = {}^{x_1 \cdots x_m} \mathbf{x} \left(P - \sum_i^{1|m} k_i \frac{x_1 \cdots x_i}{x_1 \cdots x_m} \mathbf{x} \right)^2 - \sum_{i < j}^{1|m} k_i \cdot k_j \left({}^{x_1 \cdots x_j / x_1 \cdots x_i} \mathbf{x} + \frac{{}^{x_1 \cdots x_j / x_1 \cdots x_i} \mathbf{x}^2}{x_1 \cdots x_m} \mathbf{x} \right) \\ & K_z \mathbf{x} \exp \left(k_i^\mu \eta X_{-\mathbb{N}}^\nu w_i^n \right) \exp \left(-k_i^\mu \eta X_\mathbb{N}^\nu w_i^{-n} \right) y^n K_z \end{aligned}$$