

initial state=infinite past $\tau \curvearrowleft +\infty i \Rightarrow x = {}^{\tau i} \mathbf{e} \curvearrowleft {}^{-\infty} \mathbf{e} = +0$

final state=infinite future $\tau \curvearrowright -\infty i \Rightarrow x = {}^{\tau i} \mathbf{e} \curvearrowright {}^{+\infty} \mathbf{e} = +\infty$

Faddeev-Popov gauge fixing

$1 \leq i < j < k \leq m: \infty > u_i > u_j > u_k > 0$ radial coord

$$\infty > u_1 \geq u_{i-1} > u_i > u_{i+1} \geq u_{j-1} > u_j > u_{j+1} \geq u_{k-1} > u_k > u_{k+1} \geq u_m > 0$$

$$\int_{\Lambda_{u_\ell \neq ijk}^{u_i u_j u_k}}$$

$$\begin{cases} \infty > u_1 \geq u_{i-1} > u_i \\ u_i > u_{i+1} \geq u_{j-1} > u_j \\ u_j > u_{j+1} \geq u_{k-1} > u_k \\ u_k > u_{k+1} \geq u_m > 0 \end{cases}$$

$$= (u_i - u_j) (u_i - u_k) (u_j - u_k) \int$$

$$\begin{cases} du_1 \cdots du_{i-1} \\ du_{i+1} \cdots du_{j-1} \\ du_{j+1} \cdots du_{k-1} \\ du_{k+1} \cdots du_m \end{cases}$$

$$\infty > u_1 \geq u_{i-1} > u_i \quad u_j > u_{j+1} \geq u_{k-1} > u_k \quad u_k > u_{k+1} \geq u_m > 0$$

$$\int_{du_1 \cdots du_{i-1}} \int_{du_{j+1} \cdots du_{k-1}} \int_{du_{k+1} \cdots du_m}$$

$$\frac{u_{\ell \neq ijk} \times g}{u_i u_j u_k} \widehat{V_1 \cdots V_m} = \frac{u_1 V_1}{u_1} \cdots \frac{u_m V_m}{u_m}$$

$$\frac{u_{\ell \neq ijk} \times g}{u_i \times g u_j \times g u_k \times g} \widehat{V_1 \cdots V_m} = \prod_{\ell} \frac{-2}{a - c u_{\ell} \times g} \frac{u_{\ell \neq ijk} \times g}{u_i u_j u_k} \widehat{V_1 \cdots V_m}$$

$$\Lambda_{u_\ell \neq ijk}^{u_i \rtimes gu_j \rtimes gu_k \rtimes g} = \prod_\ell \overbrace{a - cu_\ell \rtimes g}^2 d\,\Lambda_{u_\ell \neq ijk}^{u_i u_j u_k}$$

$$\Lambda_{u_\ell \neq ijk}^{u_i \rtimes gu_j \rtimes gu_k \rtimes g} \overset{u_\ell \neq ijk \rtimes g}{=} \widehat{V_1 .. V_m} = \Lambda_{u_\ell \neq ijk}^{u_i u_j u_k} \overset{u_\ell \neq ijk}{=} \widehat{V_1 .. V_m}_{u_i u_j u_k}$$

$$\underline{L-1}\,\Phi_1=0=\underline{L-1}\,\Phi_m$$

$$\left\{ \begin{array}{l} \Phi_1^{\infty \rightsquigarrow^{u_1}} |\Omega| u_1^{u_1} V_1 \\ \frac{u_m V_m}{u_m} \Omega \rightsquigarrow_0 \Phi_m \end{array} \right. \Rightarrow \Lambda^{u_1 1 u_m} \underbrace{V_1 \cdots V_m}_{u_1 1 u_m} \rightsquigarrow_{u_m}^{\infty \rightsquigarrow^{u_1}_0} \Phi_1 | V_2 \Delta V \Delta V \cdots \Delta_{m-1} V_{m-1} \Phi_m$$

$$\infty > u_1 > u_2 = 1 > u_3 > \cdots > u_{m-1} > u_m > 0$$

$$3 \leq i \leq m-1: \quad 1 > x_i = u_i / u_{i-1} > 0$$

$$\Lambda^{u_1 1 u_m} \underbrace{V_1 \cdots V_m}_{u_1 1 u_m} = \underbrace{u_1 - 1}_{\rightsquigarrow 1} \underbrace{u_1 - u_m}_{\rightsquigarrow 1} \underbrace{1 - u_m}_{\rightsquigarrow 1} \int_{d u_3 \cdots d u_{m-1}}^{1 > u_3 > \cdots > u_{m-1} > 0} \Omega | \frac{u_1 V_1}{u_1} \frac{V_2}{1} \frac{u_3 V_3}{u_3} \cdots \frac{u_{m-1} V_{m-1}}{u_{m-1}} \frac{u_m V_m}{u_m} \Omega$$

$$= \underbrace{1 - 1/u_1}_{\rightsquigarrow 1} \underbrace{1 - u_m/u_1}_{\rightsquigarrow 1} \underbrace{1 - u_m}_{\rightsquigarrow 1} \underbrace{\Omega | u_1^{u_1} V_1}_{\rightsquigarrow \Phi_1} \int_{d u_3 / u_3 \cdots d u_{m-1} / u_{m-1}}^{1 > u_3 > \cdots > u_{m-1} > 0} V_2^{u_3 V_3} \cdots V_{m-1}^{u_{m-1} V_{m-1}} \underbrace{u_m V_m \Omega / u_m}_{\rightsquigarrow \Phi_m}$$

$$\rightsquigarrow \int_{d u_3 / u_3 \cdots d u_{m-1} / u_{m-1}}^{1 > u_3 > \cdots > u_{m-1} > 0} \Phi_1 | V_2^{u_3 V_3} \cdots V_{m-1}^{u_{m-1} V_{m-1}} \Phi_m = \int_{d x_3 / x_3}^{0|1} \cdots \int_{d x_{m-1} / x_{m-1}}^{0|1} \Phi_1 | V_2^{x_3 V_3} \cdots V_{m-1}^{x_{m-1} V_{m-1}} \Phi_m$$

$$= \int_{d x_3 / x_3}^{0|1} \cdots \int_{d x_{m-1} / x_{m-1}}^{0|1} \Phi_1 | V_2^{x_3^L V_3} x_3^{-L} \widehat{x_3 x_4}^L V_4 \widehat{x_3 x_4}^{-L} \cdots \widehat{x_3 \cdots x_{m-1}}^L V_{m-1} \widehat{x_3 \cdots x_{m-1}}^{-L} \Phi_m$$

$$= \int_{d x_3 / x_3}^{0|1} \cdots \int_{d x_{m-1} / x_{m-1}}^{0|1} \Phi_1 | V_2^{x_3^L V_3} x_3^L V_4^L \cdots x_{m-1}^L V_{m-1} \widehat{x_3 \cdots x_{m-1}}^{-1} \Phi_m$$

$$= \int_{d x_3 / x_3}^{0|1} \cdots \int_{d x_{m-1} / x_{m-1}}^{0|1} \Phi_1 | V_2^{x_3^{L-1} V_3} x_3^{L-1} V_4^{L-1} \cdots x_{m-1}^{L-1} V_{m-1} \Phi_m = \Phi_1 | V_2 \Delta V_3 \Delta V \cdots \Delta_{m-1} V_{m-1} \Phi_m$$