

$$\bigvee_{o < e} \bigwedge_{\mathbb{Y}} o \leqslant \mathbb{Y} \leqslant e$$

$$\bigwedge_{\mathbb{Y}} \bigvee_{\bar{\mathbb{Y}}} \begin{cases} \mathbb{Y} \wedge \bar{\mathbb{Y}} = o \\ \mathbb{Y} \vee \bar{\mathbb{Y}} = e \end{cases}$$

$$\mathbb{Y} \wedge \underline{\mathbb{Y} \vee \bar{\mathbb{Y}}} \stackrel{D1}{=} \underline{\mathbb{Y} \wedge \mathbb{Y}} \vee \underline{\mathbb{Y} \wedge \bar{\mathbb{Y}}} \Leftrightarrow \mathbb{Y} \vee \underline{\mathbb{Y} \wedge \bar{\mathbb{Y}}} = \underline{\mathbb{Y} \vee \mathbb{Y}} \wedge \underline{\mathbb{Y} \vee \bar{\mathbb{Y}}}$$

$$\Rightarrow : \mathbb{Y} \vee \underline{\mathbb{Y} \wedge \bar{\mathbb{Y}}} \stackrel{\text{abs}}{=} \overbrace{\mathbb{Y} \vee \underline{\mathbb{Y} \wedge \bar{\mathbb{Y}}}} \vee \underline{\mathbb{Y} \wedge \bar{\mathbb{Y}}} \stackrel{\text{ass}}{=} \mathbb{Y} \vee \overbrace{\underline{\mathbb{Y} \wedge \bar{\mathbb{Y}}} \vee \underline{\mathbb{Y} \wedge \bar{\mathbb{Y}}}}$$

$$\stackrel{D1}{=} \mathbb{Y} \vee \overbrace{\underline{\mathbb{Y} \vee \mathbb{Y}} \wedge \bar{\mathbb{Y}}} \stackrel{\text{abs}}{=} \overbrace{\underline{\mathbb{Y} \vee \mathbb{Y}} \wedge \bar{\mathbb{Y}}} \vee \overbrace{\underline{\mathbb{Y} \vee \bar{\mathbb{Y}}} \wedge \bar{\mathbb{Y}}} \stackrel{D1}{=} \underline{\mathbb{Y} \vee \mathbb{Y}} \wedge \underline{\mathbb{Y} \vee \bar{\mathbb{Y}}}$$

$\mathbb{Y} \wedge \bar{\mathbb{Y}}$ eind

$$\begin{cases} \mathbb{Y} \vee \tilde{\mathbb{Y}} = e \\ \mathbb{Y} \wedge \tilde{\mathbb{Y}} = o \end{cases} \Rightarrow \tilde{\mathbb{Y}} \stackrel{\text{abs}}{=} \tilde{\mathbb{Y}} \wedge \underline{\mathbb{Y} \vee \tilde{\mathbb{Y}}} = \tilde{\mathbb{Y}} \wedge \underline{\mathbb{Y} \vee \bar{\mathbb{Y}}} \stackrel{D1}{=} \tilde{\mathbb{Y}} \wedge \mathbb{Y} \vee \tilde{\mathbb{Y}} \wedge \bar{\mathbb{Y}} = \bar{\mathbb{Y}} \wedge \mathbb{Y} \vee \tilde{\mathbb{Y}} \wedge \bar{\mathbb{Y}} \stackrel{D1}{=} \bar{\mathbb{Y}} \wedge \underline{\mathbb{Y} \vee \tilde{\mathbb{Y}}} = \bar{\mathbb{Y}} \wedge e = \bar{\mathbb{Y}}$$

$$\bar{\bar{\mathbb{Y}}} = \mathbb{Y}$$

$$\bar{\bar{\mathbb{Y}}} = \bar{\bar{\mathbb{Y}}} \wedge e = \bar{\bar{\mathbb{Y}}} \wedge \underline{\bar{\mathbb{Y}} \vee \mathbb{Y}} = \bar{\bar{\mathbb{Y}}} \wedge \bar{\mathbb{Y}} \vee \bar{\bar{\mathbb{Y}}} \wedge \mathbb{Y} = o \vee \bar{\bar{\mathbb{Y}}} \wedge \mathbb{Y} = \bar{\bar{\mathbb{Y}}} \wedge \mathbb{Y} \vee \bar{\bar{\mathbb{Y}}} \wedge \mathbb{Y} = \bar{\bar{\mathbb{Y}}} \vee \bar{\bar{\mathbb{Y}}} \wedge \mathbb{Y} = e \wedge \mathbb{Y} = \mathbb{Y}$$

$$\begin{cases} \bar{o} = e \\ \bar{e} = o \end{cases}$$

$$\begin{cases} o \vee e = e \\ o \wedge e = o \end{cases} \Rightarrow \begin{cases} \bar{e} = o \\ \bar{o} = e \end{cases}$$

$$\mathbb{Y} \leq \mathbb{A} \Leftrightarrow \mathbb{Y} \wedge \bar{\mathbb{A}} = o \Leftrightarrow \bar{\mathbb{Y}} \vee \mathbb{A} = e \Leftrightarrow \bar{\mathbb{A}} \leq \bar{\mathbb{Y}}$$

$$\mathbb{Y} \leq \mathbb{A} \Rightarrow \mathbb{Y} \wedge \bar{\mathbb{A}} = \underbrace{\mathbb{Y} \wedge \mathbb{A}} \wedge \bar{\mathbb{A}} = \mathbb{Y} \wedge \underbrace{\mathbb{A} \wedge \bar{\mathbb{A}}} = \mathbb{Y} \wedge o = o$$

$$\mathbb{Y} \wedge \bar{\mathbb{A}} = o \Rightarrow \bar{\mathbb{Y}} \vee \mathbb{A} = \overline{\mathbb{Y} \wedge \bar{\mathbb{A}}} = \bar{o} = e$$

$$\mathbb{Y} \wedge \bar{\mathbb{A}} = o \Rightarrow \bar{\mathbb{A}} = \bar{\mathbb{A}} \wedge e = \bar{\mathbb{A}} \wedge \underbrace{\mathbb{Y} \vee \bar{\mathbb{Y}}} = \underbrace{\bar{\mathbb{A}} \wedge \mathbb{Y}} \vee \underbrace{\bar{\mathbb{A}} \wedge \bar{\mathbb{Y}}} = o \vee \underbrace{\bar{\mathbb{A}} \wedge \bar{\mathbb{Y}}} = \bar{\mathbb{A}} \wedge \bar{\mathbb{Y}} \Rightarrow \bar{\mathbb{A}} \leq \bar{\mathbb{Y}}$$

$$\bar{\mathbb{A}} \leq \bar{\mathbb{Y}} \Rightarrow \mathbb{Y} = \bar{\mathbb{Y}} \leq \bar{\mathbb{A}} = \mathbb{A}$$

$$\begin{cases} \overline{\mathbb{Y} \vee \mathbb{A}} = \bar{\mathbb{Y}} \wedge \bar{\mathbb{A}} \\ \overline{\mathbb{Y} \wedge \mathbb{A}} = \bar{\mathbb{Y}} \vee \bar{\mathbb{A}} \end{cases}$$

$$\begin{cases} \mathbb{Y} \vee \mathbb{A} = \sup \mathbb{Y}: \mathbb{A} \\ \mathbb{Y} \wedge \mathbb{A} = \inf \mathbb{Y}: \mathbb{A} \end{cases}$$