

$$\mathcal{G} \in \mathbb{Z} \setminus \text{ganz}$$

$$\bigvee_{q_i}^{\mathbb{G} \times \mathbb{Q}} \mathbb{G} \times \mathbb{Q} = \mathbb{G}[q_1 \cdots q_n] \Rightarrow \bigvee_c^{\mathbb{G} \vdash 0} \mathbb{G} \times \mathbb{Q} = \mathbb{G}/c^{\mathbb{N}}$$

$$\bigvee_{a_i}^{\mathbb{G}} \bigvee_{b_i}^{\mathbb{G} \vdash 0} q_i = \frac{a_i}{b_i} \Rightarrow \begin{cases} b_1 \cdots a_i \cdots b_n & \in \mathbb{G} \\ c = b_1 \cdots b_n & \in \mathbb{G} \vdash 0 \end{cases}$$

$$\frac{a_i}{b_i} = \frac{b_1 \cdots a_i \cdots b_n}{b_1 \cdots b_i \cdots b_n} \in \mathbb{G}[\frac{1}{b_1 \cdots b_n}] \Rightarrow \mathbb{G} \times \mathbb{Q} = \mathbb{G}[\frac{a_1}{b_1} \cdots \frac{a_n}{b_n}] = \mathbb{G}[\frac{1}{b_1 \cdots b_n}]$$

$$c \in \mathbb{G} \vdash 0 \quad (1) \quad c \in \bigcap_{0 \neq p \triangleleft \mathbb{G}} p: \quad (2) \quad \bigwedge_{0 \neq a \triangleleft \mathbb{G}} c^{\mathbb{N}} \cap a \neq \emptyset: \quad (3) \quad \mathbb{G} \times \mathbb{Q} = \mathbb{G}/c^{\mathbb{N}}$$

$$(1) \Rightarrow (2): \quad \mathcal{I} = \frac{0 \neq \mathbb{1} \triangleleft \mathbb{G}}{c^{\mathbb{N}} \cap \mathbb{1} = \emptyset}: \quad \nexists \mathcal{I} \neq \emptyset \xrightarrow{\text{Zorn}} \bigvee \mathfrak{p} \underset{\max}{\in} \mathcal{I}$$

\mathfrak{p} prim

$$\begin{aligned} \nexists \dot{a} \notin \mathfrak{p} \ni a \dot{a} \Rightarrow \mathfrak{p} \triangleleft \underline{p + \mathbb{G} \dot{a}} \Rightarrow \mathfrak{p} + \mathbb{G} \dot{a} \notin \mathcal{I} \Rightarrow \bigvee_{\dot{m}}^{\mathbb{N}} c^{\dot{m}} \in \mathfrak{p} + \mathbb{G} \dot{a} \Rightarrow \bigvee_{\dot{p}}^{\mathfrak{p}} \bigvee_{\dot{b}}^{\mathbb{G}} c^{\dot{m}} = \dot{p} + \dot{a} \dot{b} \\ \Rightarrow c^{m+\dot{m}} = \underline{p + ab} \dot{p} + \dot{a} \dot{b} = \underline{p\dot{p} + p\dot{a}\dot{b} + ab\dot{p}} \underset{\in \mathfrak{p}}{+} \underline{a\dot{a}b\dot{b}} \in \mathfrak{p} \nexists c^{\mathbb{N}} \cap \mathfrak{p} = \emptyset \end{aligned}$$

$$(2) \Rightarrow (3): \quad \bigwedge_b^{\mathbb{G} \vdash 0} \bigvee_n^{\mathbb{N}} c^n \in \mathbb{G} b \Rightarrow \bigvee_d^{\mathbb{G}} c^n = db \Rightarrow \frac{a}{b} = \frac{ad}{c^n} \in \mathbb{G}/c^{\mathbb{N}} \Rightarrow \mathbb{G} \times \mathbb{Q} = \mathbb{G}/c^{\mathbb{N}}$$

$$(3) \Rightarrow (1): \quad 0 \neq \mathfrak{p} \triangleleft \mathbb{G} \Rightarrow \bigvee_{p \neq 0}^{\mathfrak{p}} \bigvee_a^{\mathbb{G}} \frac{1}{p} = \frac{a}{c^n} \Rightarrow c^n = ap \in \mathfrak{p} \Rightarrow c \in \mathfrak{p}$$