

$$\mathcal{S}_J(u) = \exp \int^{dx} \mathcal{V}(\partial_x^J) \exp \int J u \exp \int \frac{JKJ}{2}$$

$$\varphi_a \text{ harm} \Rightarrow \mathcal{S}_0(\varphi_a) = \int^{\varphi \searrow 0} \exp \int \mathcal{L}_{\varphi + \varphi_a} = \int^{\psi \searrow \varphi_a} \exp \int \mathcal{L}_\psi$$

$$\begin{aligned} \int^{\varphi \searrow 0} \exp \int \overline{\mathcal{L}_{\varphi + \varphi_a} + \int \varphi + \varphi_a J} &= \int^{\psi \searrow \varphi_a} \exp \int \overline{\mathcal{L}_\psi + \int \psi J} = \int^{\psi \searrow \varphi_a} \exp \int \overline{\mathcal{L}_\psi^0 + \int \mathcal{V}(\psi) + \int \psi J} \\ &= \int^{\psi \searrow \varphi_a} \exp \int^{dx} \mathcal{V}(x\psi) \exp \int \overline{\mathcal{L}_\psi^0 + \int \psi J} = \int^{\psi \searrow \varphi_a} \exp \int^{dx} \mathcal{V}(\partial_x^J) \exp \int \overline{\mathcal{L}_\psi^0 + \int \psi J} \\ &= \exp \int^{dx} \mathcal{V}(\partial_x^J) \int^{\psi \searrow \varphi_a} \exp \int \overline{\mathcal{L}_\psi^0 + \int \psi J} = \exp \int^{dx} \mathcal{V}(\partial_x^J) \int^{\varphi \searrow 0} \exp \int \overline{\mathcal{L}_{\varphi + \varphi_a}^0 + \int \varphi + \varphi_a J} \\ &= \exp \int^{dx} \mathcal{V}(\partial_x^J) \int^{\varphi \searrow 0} \exp \int \overline{\mathcal{L}_\varphi^0 + \int \varphi J + \int \varphi_a J} = \exp \int^{dx} \mathcal{V}(\partial_x^J) \exp \int \varphi_a J \int^{\varphi \searrow 0} \exp \int \overline{\mathcal{L}_\varphi^0 + \int \varphi J} \\ &= \exp \int^{dx} \mathcal{V}(\partial_x^J) \exp \int \varphi_a J \exp \int \frac{JKJ}{2} \end{aligned}$$

$$\partial_{x_1}^\varphi \cdots \partial_{x_n}^\varphi \mathcal{S}_J(0) = {}^{x_1} \Delta \mathbf{x} \cdots \mathbf{x} {}^{x_n} \Delta \partial_{x_1}^J \cdots \partial_{x_n}^J \mathcal{W}$$

$$\begin{aligned}
& \sum_{0 \leq n} \frac{1}{n!} \int^{dx_1} x_1 \varphi \cdots \int^{dx_n} x_n \varphi \partial_{x_1}^\varphi \cdots \partial_{x_n}^\varphi \mathcal{S}_J(0) \\
&= \sum_{0 \leq n} \frac{1}{n!} \varphi \mathbf{x} \cdots \mathbf{x} \varphi d^n \mathcal{S}_J(0) \stackrel{\text{Tay}}{=} \mathcal{S}_J(\varphi) = \exp \int^dx \mathcal{V}(\partial_x^J) \exp \int \varphi J \exp \int \frac{J \overline{KJ}}{2} \\
&= \exp \int^dx \mathcal{V}(\partial_x^J) \sum_{0 \leq n} \frac{1}{n!} \overline{\int \varphi J}^n \exp \int \frac{J \overline{KJ}}{2} \\
&= \exp \int^dx \mathcal{V}(\partial_x^J) \sum_{0 \leq n} \frac{1}{n!} \int^{dx_1} x_1 \varphi {}^{x_1} J \cdots \int^{dx_n} x_n \varphi {}^{x_n} J \exp \int \frac{J \overline{KJ}}{2} \\
&= \exp \int^dx \mathcal{V}(\partial_x^J) \sum_{0 \leq n} \frac{1}{n!} \int^{dx_1} x_1 \varphi \cdots \int^{dx_n} x_n \varphi {}^{x_1} \Delta \mathbf{x} \cdots \mathbf{x} {}^{x_n} \Delta \partial_{x_1}^J \cdots \partial_{x_n}^J \exp \int \frac{J \overline{KJ}}{2} \\
&= \exp \int^dx \mathcal{V}(\partial_x^J) \sum_{0 \leq n} \frac{1}{n!} \int^{dx_1} x_1 \overline{\Delta \varphi} \cdots \int^{dx_n} x_n \overline{\Delta \varphi} \partial_{x_1}^J \cdots \partial_{x_n}^J \exp \int \frac{J \overline{KJ}}{2} \\
&= \sum_{0 \leq n} \frac{1}{n!} \int^{dx_1} x_1 \overline{\Delta \varphi} \cdots \int^{dx_n} x_n \overline{\Delta \varphi} \exp \int^dx \mathcal{V}(\partial_x^J) \partial_{x_1}^J \cdots \partial_{x_n}^J \exp \int \frac{J \overline{KJ}}{2} \\
&= \sum_{0 \leq n} \frac{1}{n!} \int^{dx_1} x_1 \overline{\Delta \varphi} \cdots \int^{dx_n} x_n \overline{\Delta \varphi} \partial_{x_1}^J \cdots \partial_{x_n}^J \underbrace{\exp \int^dx \mathcal{V}(\partial_x^J) \exp \int \frac{J \overline{KJ}}{2}}_{= \mathcal{W}(J)} \\
&= \sum_{0 \leq n} \frac{1}{n!} \int^{dx_1} x_1 \overline{\Delta \varphi} \cdots \int^{dx_n} x_n \overline{\Delta \varphi} \partial_{x_1}^J \cdots \partial_{x_n}^J \mathcal{W}(J) = \sum_{0 \leq n} \frac{1}{n!} \int^{dx_1} x_1 \varphi \cdots \int^{dx_n} x_n \varphi {}^{x_1} \Delta \mathbf{x} \cdots \mathbf{x} {}^{x_n} \Delta \partial_{x_1}^J \cdots \partial_{x_n}^J \mathcal{W}(J)
\end{aligned}$$