$$\begin{array}{c|c} \acute{\mathcal{D}}_3 & 0 \\ \hline 0 & J_3\acute{\mathcal{D}}_3 J_3 \end{array}^{2} = \begin{array}{c|c} \acute{\mathcal{D}}_3^2 & 0 \\ \hline 0 & -J_3\acute{\mathcal{D}}_3^2 J_3 \end{array} = \begin{array}{c|c} \acute{\mathcal{D}}_3^2 & 0 \\ \hline 0 & J_3\acute{\mathcal{D}}_3 J_3^{-1} \end{array}$$

$$\eth\left(d+\acute{\omega}\right) = d\acute{\omega} + \acute{\omega} \boxtimes \acute{\omega} = \begin{array}{c|c|c} d\omega_a \boxtimes \begin{array}{c|c} a & 0 \\ \hline 0 & 0 \end{array} + d\omega \boxtimes \begin{array}{c|c} 2/3 & 0 \\ \hline 0 & 2 \end{array} & 0 & 0 \\ \hline 0 & d\omega_a \boxtimes a - d\omega \boxtimes 4/3 & 0 \\ \hline 0 & 0 & \eth\Omega \boxtimes \begin{array}{c|c} 1 & 0 \\ \hline 0 & 1 \end{array} + d\omega_a \boxtimes \begin{array}{c|c} a & 0 \\ \hline 0 & 0 \end{array} - d\omega \boxtimes \begin{array}{c} 1/3 \\ \hline 0 \end{array} \end{array}$$

$$\left(\sigma_1 \boxtimes M_1 + \sigma \boxtimes N\right) \divideontimes \left(\sigma_2 \boxtimes M_2 + \sigma \boxtimes N\right) = \sigma_1 \boxtimes M_1 \divideontimes \sigma_2 \boxtimes M_2 + \operatorname{tr} N^* N \sigma \divideontimes \sigma \Leftarrow \operatorname{tr} M_1^* N = 0 = \operatorname{tr} N^* M_2$$

$$\text{LHS} = \operatorname{tr}\left(\sigma_1 \boxtimes M_1 + \sigma \boxtimes N\right)^* \left(\sigma_2 \boxtimes M_2 + \sigma \boxtimes N\right) = \operatorname{tr}\left(\sigma_1^*\sigma_2 \boxtimes M_1^* M_2 + \sigma_1^*\sigma \boxtimes M_1^* N + \sigma^*\sigma_2 \boxtimes N^* M_2 + \sigma^*\sigma \boxtimes N^* N\right)$$

$$= \sigma_1 \divideontimes \sigma_2 \operatorname{tr} M_1^* M_2 + \sigma_1 \divideontimes \sigma \operatorname{tr} M_1^* N + \sigma \divideontimes \sigma_2 \operatorname{tr} N^* M_2 + \sigma \divideontimes \sigma \operatorname{tr} N^* N = \text{RHS}$$

$$\lceil \eth\left(d+\acute{\omega}\right) \rceil^{2} = \operatorname{tr}_{\mathbb{G}} 4 \lceil d\omega_a \boxtimes a \rceil^{2} + \operatorname{tr}_{\mathbb{G}} 4 \lceil \eth\Omega \rceil^{2} + \operatorname{tr}_{\mathbb{G}} \frac{40}{3} \lceil d\omega \rceil^{2}$$

$$\text{LHS} = \lceil d\omega_a \boxtimes a \rceil^{2} + \left(4 + 3\frac{4}{9}\right) \lceil d\omega \rceil^{2} + \lceil d\omega_a \boxtimes a \rceil^{2} + 3\frac{16}{9} \lceil d\omega \rceil^{2} + 4 \lceil \eth\Omega \rceil^{2} + 2 \lceil d\omega_a \boxtimes a \rceil^{2} + 2\left(1 + 3\frac{1}{9}\right) \lceil d\omega \rceil^{2}$$

$$\daleth_\mu \times \acute{\mathcal{M}}$$

$$\begin{array}{c|c|c} 0 & 0 & \varkappa\left(\partial_\mu\left(\varphi+\varepsilon\right)\right)^* \boxtimes \begin{array}{c|c} d^* & 0 \\ \hline 0 & e^* \end{array} \\ \hline 0 & 0 & \varkappa\left(\daleth_\mu\left(\varphi+\varepsilon\right)\right)^* \boxtimes \begin{array}{c|c} u^* & 0 \end{array} \\ \hline \partial_\mu\left(\varphi+\varepsilon\right) \boxtimes \begin{array}{c|c} d & 0 \\ \hline 0 & e \end{array} & \daleth_\mu\left(\varphi+\varepsilon\right) \boxtimes \begin{array}{c} u \\ \hline 0 \end{array} & 0 \end{array}$$

$$\daleth_\mu \varphi = \partial_\mu \varphi + \Omega_\mu \varphi - \varphi \omega_\mu$$

$$\acute{\omega}\,\acute{\mathcal{M}} = \begin{array}{c|c|c} 0 & 0 & \begin{array}{c} \varkappa\omega_a(\varphi+\varepsilon)^* \boxtimes \begin{array}{c|c} ad^* & 0 \\ \hline 0 & 0 \end{array} + \\ \varkappa\omega(\varphi+\varepsilon)^* \boxtimes \begin{array}{c|c} 2d^*/3 & 0 \\ \hline 0 & 2e^* \end{array} \end{array} \\ \hline 0 & 0 & \begin{array}{c} \varkappa\omega_a(\varphi+\varepsilon)^* \boxtimes \left[au^* \quad 0\right] - \\ \varkappa\omega(\varphi+\varepsilon)^* \boxtimes \left[4u^*/3 \quad 0\right] \end{array} \\ \hline \begin{array}{c} \Omega\left(\varphi+\varepsilon\right) \boxtimes \begin{array}{c|c} d & 0 \\ \hline 0 & e \end{array} + \\ \omega_a\left(\varphi+\varepsilon\right) \boxtimes \begin{array}{c|c} ad & 0 \\ \hline 0 & 0 \end{array} - \\ \omega\left(\varphi+\varepsilon\right) \boxtimes \begin{array}{c|c} d/3 & 0 \\ \hline 0 & -e \end{array} \end{array} & \begin{array}{c} \Omega\left(\varphi+\varepsilon\right) \boxtimes \begin{bmatrix} u \\ 0 \end{bmatrix} + \\ \omega_a\left(\varphi+\varepsilon\right) \boxtimes \begin{bmatrix} au \\ 0 \end{bmatrix} - \\ \omega\left(\varphi+\varepsilon\right) \boxtimes \begin{bmatrix} u/3 \\ 0 \end{bmatrix} \end{array} & 0 \end{array}$$

$$
\acute{\mathcal{M}}\,\acute{\omega} = \left[\begin{array}{c|c|c}
0 & 0 & \begin{array}{l} \varkappa(\varphi+\varepsilon)^*\Omega \bowtie \begin{array}{c|c} d^* & 0 \\ \hline 0 & e^* \end{array} + \\ \varkappa(\varphi+\varepsilon)^*\omega_a \bowtie \begin{array}{c|c} d^*a & 0 \\ \hline 0 & 0 \end{array} - \\ \varkappa(\varphi+\varepsilon)^*\omega \bowtie \begin{array}{c|c} d^*/3 & 0 \\ \hline 0 & -e^* \end{array} \end{array} \\
\hline
0 & 0 & \begin{array}{l} \varkappa(\varphi+\varepsilon)^*\Omega \bowtie \begin{bmatrix} u^* & 0 \end{bmatrix} + \\ \varkappa(\varphi+\varepsilon)^*\omega_a \bowtie \begin{bmatrix} u^*a & 0 \end{bmatrix} - \\ \varkappa(\varphi+\varepsilon)^*\omega \bowtie \begin{bmatrix} u^*/3 & 0 \end{bmatrix} \end{array} \\
\hline
\begin{array}{l} (\varphi+\varepsilon)\,\omega_a \bowtie \begin{array}{c|c} da & 0 \\ \hline 0 & 0 \end{array} + \\ (\varphi+\varepsilon)\,\omega \bowtie \begin{array}{c|c} 2d/3 & 0 \\ \hline 0 & 2e \end{array} \end{array} & \begin{array}{l} (\varphi+\varepsilon)\,\omega_a \bowtie \begin{bmatrix} ua \\ 0 \end{bmatrix} - \\ (\varphi+\varepsilon)\,\omega \bowtie \begin{bmatrix} 4u/3 \\ 0 \end{bmatrix} \end{array} & 0
\end{array}\right]
$$

$$
\acute{\omega} \times \acute{\mathcal{M}} = \left[\begin{array}{c|c|c}
0 & 0 & \varkappa\left(\omega(\varphi+\varepsilon)^* - (\varphi+\varepsilon)^*\Omega\right) \bowtie \begin{array}{c|c} d^* & 0 \\ \hline 0 & e^* \end{array} \\
\hline
0 & 0 & -\varkappa\left((\omega(\varphi+\varepsilon))^* + ((\varphi+\varepsilon)\,\Omega)^*\right) \bowtie \left[u^* \right. \\
\hline
(\Omega(\varphi+\varepsilon) - (\varphi+\varepsilon)\,\omega) \bowtie \begin{array}{c|c} d & 0 \\ \hline 0 & e \end{array} & (\Omega(\varphi+\varepsilon) + (\varphi+\varepsilon)\,\omega) \bowtie \begin{bmatrix} u \\ 0 \end{bmatrix} & 0
\end{array}\right]
$$

$$
\nabla_\mu \times \acute{\mathcal{M}} = \partial_\mu \acute{\mathcal{M}} + \acute{\omega}_\mu \times \acute{\mathcal{M}} =
$$

$$
\left[\begin{array}{c|c|c}
0 & 0 & \varkappa\left(\partial_\mu\varphi^* + \omega_\mu(\varphi+\varepsilon)^* - (\varphi+\varepsilon)^*\Omega_\mu\right) \bowtie \begin{array}{c|c} d^* & 0 \\ \hline 0 & e^* \end{array} \\
\hline
0 & 0 & \varkappa\left(\partial_\mu\varphi^* - \omega_\mu(\varphi+\varepsilon)^* - (\varphi+\varepsilon)^*\Omega_\mu\right) \bowtie \begin{bmatrix} u^* & 0 \end{bmatrix} \\
\hline
\left(\partial_\mu\varphi + \Omega_\mu(\varphi+\varepsilon) - (\varphi+\varepsilon)\,\omega_\mu\right) \bowtie \begin{array}{c|c} d & 0 \\ \hline 0 & e \end{array} & \left(\partial_\mu\varphi + \Omega_\mu(\varphi+\varepsilon) + (\varphi+\varepsilon)\,\omega_\mu\right) \bowtie \begin{bmatrix} u \\ 0 \end{bmatrix} & 0
\end{array}\right]
$$

$$
\nabla_\alpha \times \acute{\mathcal{M}}\, \nabla_\mu \times \acute{\mathcal{M}} / \varkappa =
$$

$$
\left[\begin{array}{c|c|c}
(\nabla_\alpha(\varphi+\varepsilon))^* \nabla_\mu(\varphi+\varepsilon) \bowtie \begin{array}{c|c} d^*d & 0 \\ \hline 0 & e^*e \end{array} & (\nabla_\alpha(\varphi+\varepsilon))^* \nabla_\mu(\varphi+\varepsilon) \bowtie \begin{bmatrix} d^*u \\ 0 \end{bmatrix} & 0 \\
\hline
(\nabla_\alpha(\varphi+\varepsilon))^* \nabla_\mu(\varphi+\varepsilon) \bowtie \begin{bmatrix} u^*d & 0 \end{bmatrix} & (\nabla_\alpha(\varphi+\varepsilon))^* \nabla_\mu(\varphi+\varepsilon) \bowtie u^*u & 0 \\
\hline
0 & 0 & \begin{array}{l} \nabla_\alpha(\varphi+\varepsilon)\,(\nabla_\mu(\varphi+\varepsilon))^* \bowtie \begin{array}{c|c} dd^* & 0 \\ \hline 0 & ee^* \end{array} \\ + \nabla_\alpha(\varphi+\varepsilon)\,(\nabla_\mu(\varphi+\varepsilon))^* \bowtie \begin{array}{c|c} uu^* & 0 \\ \hline 0 & 0 \end{array} \end{array}
\end{array}\right]
$$

$$\acute{\mathcal{M}}^2 = \left[\begin{array}{c|c|c} |\varphi+\varepsilon|^2 \otimes \begin{array}{c|c} d^* d & 0 \\ \hline 0 & e^* e \end{array} & 0 & 0 \\ \hline 0 & |\varphi+\varepsilon|^2 \otimes u^* u & 0 \\ \hline 0 & 0 & (\varphi+\varepsilon)(\varphi+\varepsilon)^* \otimes \begin{array}{c|c} dd^* & 0 \\ \hline 0 & ee^* \end{array} + (\varphi+\varepsilon)(\varphi+\varepsilon)^* \otimes \begin{array}{c|c} uu^* & 0 \\ \hline 0 & 0 \end{array} \end{array}\right]$$

$$\varphi^*\varphi = 0 = \varphi^*\varphi\varphi^*\varphi = |\varphi|^2 = \varphi^*\varphi$$

$$\mathrm{tr}\ \acute{\mathcal{M}}^2 = 2\varkappa\left(3(|d|^2+|u|^2)+|e|^2\right)|\varphi+\varepsilon|^2 = 2r\varkappa|\varphi+\varepsilon|^2$$

$$\acute{\mathcal{M}}^4 = |\varphi+\varepsilon|^2 \left[\begin{array}{c|c|c} |\varphi+\varepsilon|^2 \otimes \begin{array}{c|c} |d^* d|^2 & 0 \\ \hline 0 & |e^* e|^2 \end{array} & 0 & 0 \\ \hline 0 & |\varphi+\varepsilon|^2 \otimes |u^* u|^2 & 0 \\ \hline 0 & 0 & (\varphi+\varepsilon)(\varphi+\varepsilon)^* \otimes \begin{array}{c|c} |dd^*|^2 & 0 \\ \hline 0 & |ee^*|^2 \end{array} + (\varphi+\varepsilon)(\varphi+\varepsilon)^* \otimes \begin{array}{c|c} |uu^*|^2 & 0 \\ \hline 0 & 0 \end{array} \end{array}\right]$$

$$\varphi\varphi^*\varphi\varphi^* = |\varphi|^2\varphi\varphi^*: \quad \varphi\varphi^*\varphi\varphi^* = |\varphi|^2\varphi\varphi^*: \quad \varphi\varphi^*\varphi\varphi^* = 0 = \varphi\varphi^*\varphi\varphi^*$$

$$\mathrm{tr}\ \acute{\mathcal{M}}^4 = 2\left(3(|d|^4+|u|^4)+|e|^4\right)|\varphi+\varepsilon|^4: \quad \mathrm{tr}\ \varphi^*\varphi = \varphi^*\varphi = \mathrm{tr}\ \varphi^*\varphi: \quad \mathrm{tr}\ \acute{\mathcal{V}} = \frac{15}{4}R - 2r\varkappa|\varphi+\varepsilon|^2$$

$$-\acute{\mathcal{D}}^2 = -\ \mathrm{tr}\ \frac{3}{4}C \ast C + \frac{2}{3}|d|^2 + 4\varkappa\left(3(|d|^2+|u|^2)+|e|^2\right)|\partial(\varphi+\varepsilon)|^2 - \frac{2}{3}\varkappa\left(3(|d|^2+|u|^2)+|e|^2\right)R|\varphi+\varepsilon|^2 + 4\left(3(|d|^4+|u|^4)+|e|^4\right)|\varphi+\varepsilon|^4$$

$$90\ \mathrm{LHS} = \frac{360}{\nearrow mS}$$

$$\mathrm{tr}\ id\left(5R^2 - 2r \ast r + 2\acute{\omega} \ast \acute{\omega}\right) + 120\varkappa\left(3(|d|^2+|u|^2)+|e|^2\right)R|\varphi+\varepsilon|^2 - 60\frac{R^2}{4}\mathrm{tr}1$$

$$+360\left(3(|d|^4+|u|^4)+|e|^4\right)|\varphi+\varepsilon|^4 - 180\varkappa\left(3(|d|^2+|u|^2)+|e|^2\right)R|\varphi+\varepsilon|^2 + 180\frac{R^2}{16}\mathrm{tr}1$$

$$+360\varkappa\left(3(|d|^2+|u|^2)+|e|^2\right)|\partial(\varphi+\varepsilon)|^2 + 90|d|^2 + 30\left(-\frac{1}{8}\acute{\omega} \ast \acute{\omega}\_\mathrm{tr}|d|^2\right)$$

$$= 60|d|^2 + 360\left(3(|d|^4+|u|^4)+|e|^4\right)|\varphi+\varepsilon|^4 + 360\varkappa\left(3(|d|^2+|u|^2)+|e|^2\right)|\partial(\varphi+\varepsilon)|^2$$

$$-60\varkappa\left(3(|d|^2+|u|^2)+|e|^2\right)R|\varphi+\varepsilon|^2 + \frac{1}{4}\mathrm{tr}15\left(5R^2 - 8r \ast r - 7\acute{\omega} \ast \acute{\omega}\right)$$

$$5R^2 - 8r \ast r - 7\acute{\omega} \ast \acute{\omega} = -18\left(\acute{\omega} \ast \acute{\omega} - 2r \ast r + R^2/3\right) + 11\left(\acute{\omega} \ast \acute{\omega} - 4r \ast r + r^2\right) = -18C \ast C + 11\chi$$

$$(4\pi)^2\ \mathrm{Tr}\ \chi\left(-\acute{\mathcal{D}}^2\right) = \Lambda^4\chi_0\mathrm{tr}60$$

$$+\Lambda^2\,\chi_2\left(8\varkappa\,\underline{3\,\overbrace{d^2+u^2}+e^2}\,\overline{\varphi+\varepsilon}^{\,2}-\ \mathrm{tr}\ 5R\right)+\ \chi_4\left(4\,\underline{3\,\overbrace{d^4+u^4}+e^4}\,\overline{\varphi+\varepsilon}^{\,4}+\frac{2}{3}\overline{d}^{\,2}-\frac{3}{4}\ \mathrm{tr}\ C\maltese C\right)$$

$$+4\varkappa\,\underline{3\,\overbrace{d^2+u^2}+e^2}\ \overline{\eth\,(\varphi+\varepsilon)}^{\,2}-\frac{2}{3}\varkappa R\,\overline{\varphi+\varepsilon}^{\,2}\ \underline{3\,\overbrace{d^2+u^2}+e^2}$$