$$\overbrace{{}_g\dashv\boxtimes 1+1\boxtimes\dashv}^{2} = -\overbrace{{}_g\dashv\boxtimes 1+1\boxtimes\dashv}^{*}\underbrace{{}_g\dashv\boxtimes 1+1\boxtimes\dashv} + \frac{1}{2}\underline{\mathfrak{V}}^{\mu}\,\underline{\mathfrak{V}}^{\nu}\boxtimes\underbrace{d\dashv}_{\mu\mathfrak{l}\,\nu\mathfrak{l}} - \frac{1}{4}{}^{i}\left(d\underbrace{d^{\mathfrak{l}}\!\cdot\!\mathfrak{b}g}\right)^{j}_{ij}$$

$$\text{LHS} = {}_g\dashv^{2}\boxtimes 1+\left({}_g\dashv\underline{\mathfrak{V}}^{\mu}+\underline{\mathfrak{V}}^{\mu}{}_g\dashv\right)\boxtimes\dashv_{\mu\mathfrak{l}}+\underline{\mathfrak{V}}^{\mu}\,\underline{\mathfrak{V}}^{\nu}\boxtimes\dashv_{\mu\mathfrak{l}}\dashv_{\nu\mathfrak{l}} = -{}_g\dashv^{*}{}_g\dashv+\frac{1}{4}{}^{i}\left(dd^{\mathfrak{l}}\!\cdot\!\mathfrak{b}g\right)^{j}_{ij}\boxtimes 1+\underline{\mathfrak{V}}^{\nu}\bowtie\underline{\mathfrak{V}}^{\lambda}{}_{\nu}\left(d^{\mathfrak{l}}\!\cdot\!\mathfrak{l}g\right)$$

$$+2\,\underline{\mathfrak{V}}^{\nu}\bowtie\underline{\mathfrak{V}}^{\mu}\,{}_g\dashv_{\nu\mathfrak{l}}\boxtimes\dashv_{\mu\mathfrak{l}}+\left(\frac{\underline{\mathfrak{V}}^{\mu}\underline{\mathfrak{V}}^{\nu}+\underline{\mathfrak{V}}^{\nu}\underline{\mathfrak{V}}^{\mu}}{2}+\frac{\underline{\mathfrak{V}}^{\nu}\underline{\mathfrak{V}}^{\mu}-\underline{\mathfrak{V}}^{\mu}\underline{\mathfrak{V}}^{\nu}}{2}\right)\boxtimes\dashv_{\nu\mathfrak{l}}\dashv_{\mu\mathfrak{l}} = -\left({}_g\overset{*}{\dashv}\right)\left({}_g\dashv\right)\boxtimes 1+2\,\underline{\mathfrak{V}}^{\nu}\bowtie\underline{\mathfrak{V}}^{\mu}\,{}_g\dashv_{\nu\mathfrak{l}}\boxtimes\dashv_{\mu}$$

$$+\frac{1}{2}\underline{\mathfrak{V}}^{\mu}\,\underline{\mathfrak{V}}^{\nu}\boxtimes\overbrace{\dashv_{\nu\mathfrak{l}}\dashv_{\mu\mathfrak{l}}-\dashv_{\mu\mathfrak{l}}\dashv_{\nu\mathfrak{l}}}+1\boxtimes\underline{\mathfrak{V}}^{\mu}\bowtie\underline{\mathfrak{V}}^{\nu}\overbrace{\dashv_{\nu\mathfrak{l}}\dashv_{\mu\mathfrak{l}}+{}_{\mu}\left(d^{\mathfrak{l}}\!\cdot\!\mathfrak{l}g\right)^{\lambda}_{\nu}\dashv_{\lambda\mathfrak{l}}}-\frac{1}{4}{}^{i}\left(dd^{\mathfrak{l}}\!\cdot\!\mathfrak{b}g\right)^{j}_{ij}$$

$$\underbrace{\underline{\mathfrak{V}}^{\mu}\bowtie\underline{\mathfrak{V}}^{\alpha}}\underbrace{\underline{\mathfrak{V}}^{\mu}\bowtie\underline{\mathfrak{V}}^{\beta}}\underbrace{\overbrace{d_g\dashv\boxtimes 1+1\boxtimes\dashv}_{\alpha\mathfrak{l}\,\beta\mathfrak{l}}\overbrace{d_g\dashv\boxtimes 1+1\boxtimes\dashv}_{\mu\mathfrak{l}\,\nu\mathfrak{l}}}_{S\boxtimes\mathrm{J}} = -{}_{.}\left(d^{\mathfrak{l}}\!\cdot\!\mathfrak{l}g\right)^{.}\bowtie{}_{.}\left(d^{\mathfrak{l}}\!\cdot\!\mathfrak{l}g\right)^{.}\underbrace{1/8}_{\mathrm{J}}-\overline{d\dashv}^{2}$$

$$\underbrace{d_g\dashv\boxtimes 1+1\boxtimes\dashv}_{\alpha\mathfrak{l}\,\beta\mathfrak{l}}\underbrace{d_g\dashv\boxtimes 1+1\boxtimes\dashv}_{\mu\mathfrak{l}\,\nu\mathfrak{l}} = \overbrace{{}_{.}\underbrace{d^{\mathfrak{l}}\!\cdot\!\mathfrak{l}g}_{\alpha\beta}^{.}\boxtimes 1+1\boxtimes\underbrace{d\dashv}_{\alpha\mathfrak{l}\,\beta\mathfrak{l}}}\overbrace{{}_{.}\underbrace{d^{\mathfrak{l}}\!\cdot\!\mathfrak{l}g}_{\mu\nu}^{.}\boxtimes 1+1\boxtimes\underbrace{d\dashv}_{\mu\mathfrak{l}\,\nu\mathfrak{l}}}$$

$$= {}_{.}\underbrace{d^{\mathfrak{l}}\!\cdot\!\mathfrak{l}g}_{\alpha\beta}^{.}\,{}_{.}\underbrace{d^{\mathfrak{l}}\!\cdot\!\mathfrak{l}g}_{\mu\nu}^{.}\boxtimes 1+{}_{.}\underbrace{d^{\mathfrak{l}}\!\cdot\!\mathfrak{l}g}_{\alpha\beta}^{.}\boxtimes\underbrace{d\dashv}_{\mu\mathfrak{l}\,\nu\mathfrak{l}}+{}_{.}\underbrace{d^{\mathfrak{l}}\!\cdot\!\mathfrak{l}g}_{\mu\nu}^{.}\boxtimes\underbrace{d\dashv}_{\alpha\mathfrak{l}\,\beta\mathfrak{l}}+1\boxtimes\underbrace{d\dashv}_{\alpha\mathfrak{l}\,\beta\mathfrak{l}}\underbrace{d\dashv}_{\mu\mathfrak{l}\,\nu\mathfrak{l}}$$

$$\underbrace{\overbrace{d_g\dashv\boxtimes 1+1\boxtimes\dashv}_{\alpha\mathfrak{l}\,\beta\mathfrak{l}}\overbrace{d_g\dashv\boxtimes 1+1\boxtimes\dashv}_{\mu\mathfrak{l}\,\nu\mathfrak{l}}}_{S\boxtimes\mathrm{J}} = \underbrace{{}_{.}\underbrace{d\left(d^{\mathfrak{l}}\!\cdot\!\mathfrak{l}g\right)^{.}}_{\alpha\beta}}_{S}\underbrace{{}_{.}\underbrace{d\left(d^{\mathfrak{l}}\!\cdot\!\mathfrak{l}g\right)^{.}}_{\mu\nu}}_{\mathrm{J}}+\underbrace{\underbrace{d\dashv}_{\alpha\mathfrak{l}\,\beta\mathfrak{l}}\underbrace{d\dashv}_{\mu\mathfrak{l}\,\nu\mathfrak{l}}}_{\mathrm{J}}$$

$$+\underbrace{{}_{.}\underbrace{d^{\mathfrak{l}}\!\cdot\!\mathfrak{l}g}_{\alpha\beta}^{.}}_{S}\underbrace{\underbrace{d\dashv}_{\mu\mathfrak{l}\,\nu\mathfrak{l}}}_{\mathrm{J}}+\underbrace{{}_{.}\underbrace{d^{\mathfrak{l}}\!\cdot\!\mathfrak{l}g}_{\mu\nu}^{.}}_{S}\underbrace{\underbrace{d\dashv}_{\alpha\mathfrak{l}\,\beta\mathfrak{l}}}_{\mathrm{J}}$$

$$= -{}_{.}\underbrace{d^{\mathfrak{l}}\!\cdot\!\mathfrak{l}g}_{\alpha\beta}^{.}\bowtie{}_{.}\left(d^{\mathfrak{l}}\!\cdot\!\mathfrak{l}g\right)^{.}_{\mu\nu}\underbrace{1/8}_{\mathrm{J}}-\underbrace{\underbrace{d\dashv}^{*}_{\alpha\mathfrak{l}\,\beta\mathfrak{l}}\underbrace{d\dashv}_{\mu\mathfrak{l}\,\nu\mathfrak{l}}}_{\lrcorner} = -{}_{.}\underbrace{d^{\mathfrak{l}}\!\cdot\!\mathfrak{l}g}_{\alpha\beta}^{.}\bowtie{}_{.}\underbrace{d^{\mathfrak{l}}\!\cdot\!\mathfrak{l}g}_{\mu\nu}^{.}\underbrace{1/8}_{\lrcorner}-\underbrace{d\dashv}_{\alpha\mathfrak{l}\,\beta\mathfrak{l}}\bowtie\underbrace{d\dashv}_{\mu\mathfrak{l}\,\nu\mathfrak{l}}$$

$$-\text{LHS} = \underbrace{\underline{\mathfrak{V}}^{\mu}\bowtie\underline{\mathfrak{V}}^{\alpha}}\underbrace{\underline{\mathfrak{V}}^{\mu}\bowtie\underline{\mathfrak{V}}^{\beta}}\overbrace{{}_{.}\underbrace{d^{\mathfrak{l}}\!\cdot\!\mathfrak{l}g}_{\alpha\beta}^{.}\bowtie{}_{.}\underbrace{d^{\mathfrak{l}}\!\cdot\!\mathfrak{l}g}_{\mu\nu}^{.}\underbrace{1/8}_{\lrcorner}+\underbrace{d\dashv}_{\alpha\beta}\bowtie\underbrace{d\dashv}_{\mu\nu}}$$

$$= {}_{.}\underbrace{d^{\mathfrak{l}}\!\cdot\!\mathfrak{l}g}^{\cdot\mu\nu}\bowtie{}_{.}\underbrace{d^{\mathfrak{l}}\!\cdot\!\mathfrak{l}g}_{\mu\nu}^{.}\underbrace{1/8}_{\lrcorner}+\overbrace{d\dashv}^{\mu\nu}\bowtie\underbrace{d\dashv}_{\mu\nu} = -\text{RHS}$$

$$\mathcal{D}_{A}^{2}\Psi$$

$$\mathcal{L}\left(g{:}A{:}\Psi\right)$$