$$\left(\begin{array}{c|c} \acute{\mathcal{D}}_3 & 0 \\ \hline 0 & J_3\acute{\mathcal{D}}_3 J_3 \end{array}\right)^2 = \begin{array}{c|c} \acute{\mathcal{D}}_3^2 & 0 \\ \hline 0 & -J_3\acute{\mathcal{D}}_3^2 J_3 \end{array} = \begin{array}{c|c} \acute{\mathcal{D}}_3^2 & 0 \\ \hline 0 & J_3\acute{\mathcal{D}}_3 J_3^{-1} \end{array}$$

$$\eth\left(d+\acute{\omega}\right) = d\acute{\omega} + \acute{\omega} ⧗ \acute{\omega} =$$

$$\begin{array}{c|c|c} d\omega_a ⧗ \begin{array}{c|c} a & 0 \\ \hline 0 & 0 \end{array} + d\omega ⧗ \begin{array}{c|c} 2/3 & 0 \\ \hline 0 & 2 \end{array} & 0 & 0 \\ \hline 0 & d\omega_a ⧗ a - d\omega ⧗ 4/3 & 0 \\ \hline 0 & 0 & \eth\Omega ⧗ \begin{array}{c|c} 1 & 0 \\ \hline 0 & 1 \end{array} + d\omega_a ⧗ \begin{array}{c|c} a & 0 \\ \hline 0 & 0 \end{array} - d\omega ⧗ \begin{array}{c|c} 1/3 & 0 \\ \hline 0 & -1 \end{array} \end{array}$$

$$\left(\sigma_1 ⧗ M_1 + \sigma ⧗ N\right) \divideontimes \left(\sigma_2 ⧗ M_2 + \sigma ⧗ N\right) = \sigma_1 ⧗ M_1 \divideontimes \sigma_2 ⧗ M_2 + \underbrace{\overset{*}{N}N}\sigma \divideontimes \sigma \Leftarrow \underbrace{\overset{*}{M}_1 N} = 0 = \underbrace{\overset{*}{N}M_2}$$

$$\text{LHS} = \underbrace{\overbrace{\sigma_1 ⧗ M_1 + \sigma ⧗ N}^{*}\underbrace{\sigma_2 ⧗ M_2 + \sigma ⧗ N}} = \underbrace{\overbrace{\sigma_1 ⧗ M_1 + \sigma ⧗ N}^{*}\underbrace{\sigma_2 ⧗ M_2 + \sigma ⧗ N}}$$

$$= \underbrace{\overset{*}{\sigma}_1\sigma_2 ⧗ \overset{*}{M}_1 M_2 + \overset{*}{\sigma}_1\sigma ⧗ \overset{*}{M}_1 N + \overset{*}{\sigma}\sigma_2 ⧗ \overset{*}{N}M_2 + \overset{*}{\sigma}\sigma ⧗ \overset{*}{N}N}$$

$$= \sigma_1 \divideontimes \sigma_2 \underbrace{\overset{*}{M}_1 M_2} + \sigma_1 \divideontimes \sigma \underbrace{\overset{*}{M}_1 N} + \sigma \divideontimes \sigma_2 \underbrace{\overset{*}{N}M_2} + \sigma \divideontimes \sigma \underbrace{\overset{*}{N}N} = \text{RHS}$$

$$\overline{\eth\left(d+\acute{\omega}\right)}^{2} = \underbrace{4}_{\mathbb{G}}\,\overline{d\omega_a ⧗ a}^{2} + \underbrace{4}_{\mathbb{G}}\,\overline{\eth\Omega}^{2} + \underbrace{\frac{40}{3}}_{\mathbb{G}}\,\overline{d\omega}^{2}$$

$$\text{LHS} = \overline{d\omega_a ⧗ a}^{2} + \left(4 + 3\frac{4}{9}\right)\overline{d\omega}^{2} + \overline{d\omega_a ⧗ a}^{2} + 3\frac{16}{9}\overline{d\omega}^{2} + 4\,\overline{\eth\Omega}^{2} + 2\,\overline{d\omega_a ⧗ a}^{2} + 2\left(1 + 3\frac{1}{9}\right)\overline{d\omega}^{2}$$

$$\Lsh_\mu \times \acute{\mathcal{M}}$$

$$\begin{array}{c|c|c} 0 & 0 & \varkappa\left(\partial_\mu\left(\overset{*}{\varphi}+\varepsilon\right)\right) ⧗ \begin{array}{c|c} \overset{*}{d} & 0 \\ \hline 0 & \overset{*}{e} \end{array} \\ \hline 0 & 0 & \varkappa\left(\Lsh_\mu\left(\overset{*}{\varphi}+\varepsilon\right)\right) ⧗ \begin{array}{c|c} \overset{*}{u} & 0 \end{array} \\ \hline \partial_\mu\left(\varphi+\varepsilon\right) ⧗ \begin{array}{c|c} d & 0 \\ \hline 0 & e \end{array} & \Lsh_\mu\left(\varphi+\varepsilon\right) ⧗ \begin{array}{c} u \\ \hline 0 \end{array} & 0 \end{array}$$

$$\Lsh_\mu \varphi = \partial_\mu \varphi + \Omega_\mu \varphi - \varphi\,\omega_\mu$$

$$
\acute{\omega}\,\acute{\mathcal{M}} =
\begin{array}{c|c|c}
0 & 0 & \begin{array}{l} \varkappa\omega_a(\varphi \overset{*}{+} \varepsilon)\text{⧗}\begin{array}{c|c} a\overset{*}{d} & 0 \\ \hline 0 & 0 \end{array} + \\ \varkappa\omega(\varphi \overset{*}{+} \varepsilon)\text{⧗}\begin{array}{c|c} 2\overset{*}{d}/3 & 0 \\ \hline 0 & 2\overset{*}{e} \end{array} \end{array} \\ \hline
0 & 0 & \begin{array}{l} \varkappa\omega_a(\varphi \overset{*}{+} \varepsilon)\text{⧗}\begin{bmatrix} a\overset{*}{u} & 0 \end{bmatrix} - \\ \varkappa\omega(\varphi \overset{*}{+} \varepsilon)\text{⧗}\begin{bmatrix} 4\overset{*}{u}/3 & 0 \end{bmatrix} \end{array} \\ \hline
\begin{array}{l} \Omega\,(\varphi+\varepsilon)\text{⧗}\begin{array}{c|c} d & 0 \\ \hline 0 & e \end{array} + \\ \omega_a\,(\varphi+\varepsilon)\text{⧗}\begin{array}{c|c} ad & 0 \\ \hline 0 & 0 \end{array} - \\ \omega\,(\varphi+\varepsilon)\text{⧗}\begin{array}{c|c} d/3 & 0 \\ \hline 0 & -e \end{array} \end{array} & \begin{array}{l} \Omega\,(\varphi+\varepsilon)\text{⧗}\begin{bmatrix} u \\ 0 \end{bmatrix} + \\ \omega_a\,(\varphi+\varepsilon)\text{⧗}\begin{bmatrix} au \\ 0 \end{bmatrix} - \\ \omega\,(\varphi+\varepsilon)\text{⧗}\begin{bmatrix} u/3 \\ 0 \end{bmatrix} \end{array} & 0
\end{array}
$$

$$
\acute{\mathcal{M}}\,\acute{\omega} =
\begin{array}{c|c|c}
0 & 0 & \begin{array}{l} \varkappa(\varphi \overset{*}{+} \varepsilon)\Omega\text{⧗}\begin{array}{c|c} \overset{*}{d} & 0 \\ \hline 0 & \overset{*}{e} \end{array} + \\ \varkappa(\varphi \overset{*}{+} \varepsilon)\omega_a\text{⧗}\begin{array}{c|c} \overset{*}{d}a & 0 \\ \hline 0 & 0 \end{array} - \\ \varkappa(\varphi \overset{*}{+} \varepsilon)\omega\text{⧗}\begin{array}{c|c} \overset{*}{d}/3 & 0 \\ \hline 0 & -\overset{*}{e} \end{array} \end{array} \\ \hline
0 & 0 & \begin{array}{l} \varkappa(\varphi \overset{*}{+} \varepsilon)\Omega\text{⧗}\begin{bmatrix} \overset{*}{u} & 0 \end{bmatrix} + \\ \varkappa(\varphi \overset{*}{+} \varepsilon)\omega_a\text{⧗}\begin{bmatrix} \overset{*}{u}a & 0 \end{bmatrix} - \\ \varkappa(\varphi \overset{*}{+} \varepsilon)\omega\text{⧗}\begin{bmatrix} \overset{*}{u}/3 & 0 \end{bmatrix} \end{array} \\ \hline
\begin{array}{l} (\varphi+\varepsilon)\,\omega_a\text{⧗}\begin{array}{c|c} da & 0 \\ \hline 0 & 0 \end{array} + \\ (\varphi+\varepsilon)\,\omega\text{⧗}\begin{array}{c|c} 2d/3 & 0 \\ \hline 0 & 2e \end{array} \end{array} & \begin{array}{l} (\varphi+\varepsilon)\,\omega_a\text{⧗}\begin{bmatrix} ua \\ 0 \end{bmatrix} - \\ (\varphi+\varepsilon)\,\omega\text{⧗}\begin{bmatrix} 4u/3 \\ 0 \end{bmatrix} \end{array} & 0
\end{array}
$$

$$
\acute{\omega} \times \acute{\mathcal{M}} =
\begin{array}{c|c|c}
0 & 0 & \varkappa\left(\omega(\varphi \overset{*}{+} \varepsilon) - (\varphi \overset{*}{+} \varepsilon)\Omega\right)\text{⧗}\begin{array}{c|c} \overset{*}{d} & 0 \\ \hline 0 & \overset{*}{e} \end{array} \\ \hline
0 & 0 & -\varkappa\left((\omega\,(\varphi \overset{*}{+} \varepsilon)) + ((\varphi \overset{*}{+} \varepsilon)\,\Omega)\right)\text{⧗}\begin{bmatrix} \overset{*}{u} & 0 \end{bmatrix} \\ \hline
(\Omega\,(\varphi+\varepsilon) - (\varphi+\varepsilon)\,\omega)\text{⧗}\begin{array}{c|c} d & 0 \\ \hline 0 & e \end{array} & (\Omega\,(\varphi+\varepsilon) + (\varphi+\varepsilon)\,\omega)\text{⧗}\begin{bmatrix} u \\ 0 \end{bmatrix} & 0
\end{array}
$$

$$
\nabla_{\mu} \times \acute{\mathcal{M}} = \partial_\mu \acute{\mathcal{M}} + \acute{\omega}_\mu \times \acute{\mathcal{M}} =
$$

$$
\begin{array}{c|c|c}
0 & 0 & \varkappa\left(\partial_\mu\overset{*}{\varphi} + \omega_\mu(\varphi \overset{*}{+} \varepsilon) - (\varphi \overset{*}{+} \varepsilon)\Omega_\mu\right)\text{⧗}\begin{array}{c|c} \overset{*}{d} & 0 \\ \hline 0 & \overset{*}{e} \end{array} \\ \hline
0 & 0 & \varkappa\left(\partial_\mu\overset{*}{\varphi} - \omega_\mu(\varphi \overset{*}{+} \varepsilon) - (\varphi \overset{*}{+} \varepsilon)\Omega_\mu\right)\text{⧗}\begin{bmatrix} \overset{*}{u} & 0 \end{bmatrix} \\ \hline
\left(\partial_\mu\varphi + \Omega_\mu\,(\varphi+\varepsilon) - (\varphi+\varepsilon)\,\omega_\mu\right)\text{⧗}\begin{array}{c|c} d & 0 \\ \hline 0 & e \end{array} & \left(\partial_\mu\varphi + \Omega_\mu\,(\varphi+\varepsilon) + (\varphi+\varepsilon)\,\omega_\mu\right)\text{⧗}\begin{bmatrix} u \\ 0 \end{bmatrix} & 0
\end{array}
$$

$$\vdash_\alpha \times \acute{\mathcal{M}} \vdash_\mu \times \acute{\mathcal{M}} / \varkappa =$$

$$\begin{array}{c|c|c}
(\vdash_\alpha (\overset{*}{\varphi}+\varepsilon))\vdash_\mu (\varphi+\varepsilon) \bowtie \begin{array}{c|c} \overset{*}{d}d & 0 \\ \hline 0 & \overset{*}{e}e \end{array} & (\vdash_\alpha (\overset{*}{\varphi}+\varepsilon))\vdash_\mu (\varphi+\varepsilon) \bowtie \begin{bmatrix} \overset{*}{d}u \\ 0 \end{bmatrix} & 0 \\
\hline
(\vdash_\alpha (\overset{*}{\varphi}+\varepsilon))\vdash_\mu (\varphi+\varepsilon) \bowtie \begin{bmatrix} \overset{*}{u}d & 0 \end{bmatrix} & (\vdash_\alpha (\overset{*}{\varphi}+\varepsilon))\vdash_\mu (\varphi+\varepsilon) \bowtie \overset{*}{u}u & 0 \\
\hline
0 & 0 & \begin{array}{l} \vdash_\alpha (\varphi+\varepsilon)(\vdash_\mu (\overset{*}{\varphi}+\varepsilon)) \bowtie \begin{array}{c|c} d\overset{*}{d} & 0 \\ \hline 0 & e\overset{*}{e} \end{array} \\ +\vdash_\alpha (\varphi+\varepsilon)(\vdash_\mu (\overset{*}{\varphi}+\varepsilon)) \bowtie \begin{array}{c|c} u\overset{*}{u} & 0 \\ \hline 0 & 0 \end{array} \end{array}
\end{array}$$

$$\acute{\mathcal{M}}^2 = \begin{array}{c|c|c}
\overline{\varphi+\varepsilon}^{2} \bowtie \begin{array}{c|c} \overset{*}{d}d & 0 \\ \hline 0 & \overset{*}{e}e \end{array} & 0 & 0 \\
\hline
0 & \overline{\varphi+\varepsilon}^{2} \bowtie \overset{*}{u}u & 0 \\
\hline
0 & 0 & (\varphi+\varepsilon)(\varphi\overset{*}{+}\varepsilon) \bowtie \begin{array}{c|c} d\overset{*}{d} & 0 \\ \hline 0 & e\overset{*}{e} \end{array} + (\varphi+\varepsilon)(\varphi\overset{*}{+}\varepsilon) \bowtie \begin{array}{c|c} u\overset{*}{u} & 0 \\ \hline 0 & 0 \end{array}
\end{array}$$

$$\overset{*}{\varphi}\varphi = 0 = \overset{*}{\varphi}\varphi\overset{*}{\varphi}\varphi = \overline{\varphi}^{2} = \overset{*}{\varphi}\varphi$$

$$\underbrace{\acute{\mathcal{M}}^2} = 2\varkappa \underbrace{3\underbrace{d^2+u^2}+e^2} \overline{\varphi+\varepsilon}^{2} = 2r\varkappa\overline{\varphi+\varepsilon}^{2}$$

$$\acute{\mathcal{M}}^4 = \overline{\varphi+\varepsilon}^{2} \begin{array}{c|c|c}
\overline{\varphi+\varepsilon}^{2} \bowtie \begin{array}{c|c} \overset{*}{d}d^{2} & 0 \\ \hline 0 & \overset{*}{e}e^{2} \end{array} & 0 & 0 \\
\hline
0 & \overline{\varphi+\varepsilon}^{2} \bowtie \overset{*}{u}u^{2} & 0 \\
\hline
0 & 0 & (\varphi+\varepsilon)(\varphi\overset{*}{+}\varepsilon) \bowtie \begin{array}{c|c} d\overset{*}{d}^{2} & 0 \\ \hline 0 & e\overset{*}{e}^{2} \end{array} + (\varphi+\varepsilon)(\varphi\overset{*}{+}\varepsilon) \bowtie \begin{array}{c|c} u\overset{*}{u}^{2} & 0 \\ \hline 0 & 0 \end{array}
\end{array}$$

$$\varphi\overset{*}{\varphi}\varphi\overset{*}{\varphi} = \overline{\varphi}^{2}\varphi\overset{*}{\varphi}: \quad \varphi\overset{*}{\varphi}\varphi\overset{*}{\varphi} = \overline{\varphi}^{2}\varphi\overset{*}{\varphi}: \quad \varphi\overset{*}{\varphi}\varphi\overset{*}{\varphi} = 0 = \varphi\overset{*}{\varphi}\varphi\overset{*}{\varphi}$$

$$\underbrace{\acute{\mathcal{M}}^4} = 2\,\underbrace{3\underbrace{d^4+u^4}+e^4}\,\overline{\varphi+\varepsilon}^{4}: \quad \underbrace{\overset{*}{\varphi}\varphi} = \overset{*}{\varphi}\varphi = \underbrace{\overset{*}{\varphi}\varphi}: \quad \underbrace{\acute{\mathcal{V}}} = \frac{15}{4}R - 2r\varkappa\overline{\varphi+\varepsilon}^{2}$$

$$-\acute{\mathcal{D}}^2 = -\frac{3}{\underbrace{4}} C \bowtie C + \frac{2}{3}\overline{d}^{2} + 4\varkappa\underbrace{3\underbrace{d^2+u^2}+e^2}\,\overline{\eth(\varphi+\varepsilon)}^{2} - \frac{2}{3}\varkappa\underbrace{3\underbrace{d^2+u^2}+e^2}\,R\overline{\varphi+\varepsilon}^{2} + 4\underbrace{3\underbrace{d^4+u^4}+e^4}\,\overline{\varphi+\varepsilon}^{4}$$

$$90 \text{ LHS } = \frac{360}{\nearrow mS}\underbrace{id}\left(5R^2 - 2r\bowtie r + 2\acute{\omega}\bowtie\acute{\omega}\right) + 120\varkappa\underbrace{3\underbrace{d^2+u^2}+e^2}\,R\overline{\varphi+\varepsilon}^{2} - 60\frac{R^2}{4}\underbrace{1}$$

$$+360\underbrace{3\underbrace{d^4+u^4}+e^4}\,\overline{\varphi+\varepsilon}^{4} - 180\varkappa\underbrace{3\underbrace{d^2+u^2}+e^2}\,R\overline{\varphi+\varepsilon}^{2} + 180\frac{R^2}{16}\underbrace{1}$$

$$+360\varkappa\underbrace{3\underbrace{d^2+u^2}+e^2}\,\overline{\eth(\varphi+\varepsilon)}^{2} + 90\overline{d}^{2} + 30\left(-\frac{1}{8}\acute{\omega}\bowtie\acute{\omega}\underbrace{1} - \overline{d}^{2}\right)$$

$$= 60\,\overline{d}^{\,2} + 360\,\underline{3\underbrace{d^4 + u^4} + e^4}\,\overline{\varphi + \varepsilon}^{\,4} + 360\varkappa\,\underline{3\underbrace{d^2 + u^2} + e^2}\,\overline{\eth\,(\varphi + \varepsilon)}^{\,2}$$

$$-60\varkappa\,\underline{3\underbrace{d^2 + u^2} + e^2}\,R\,\overline{\varphi + \varepsilon}^{\,2} + \frac{1}{4}\,\underline{15}\left(5R^2 - 8r\bowtie r - 7\acute{\omega}\bowtie\acute{\omega}\right)$$

$$5\,R^2 - 8r\bowtie r - 7\,\acute{\omega}\bowtie\acute{\omega} = -18\left(\acute{\omega}\bowtie\acute{\omega} - 2r\bowtie r + R^2/3\right) + 11\left(\acute{\omega}\bowtie\acute{\omega} - 4r\bowtie r + r^2\right) = -18C\bowtie C + 11\chi$$

$$(4\pi)^2\,\underline{\chi\left(-\acute{\mathcal{D}}^2\right)} = \Lambda^4\,\chi_0\,\underline{60} + \Lambda^2\,\chi_2\left(8\varkappa\,\underline{3\underbrace{d^2 + u^2} + e^2}\,\overline{\varphi + \varepsilon}^{\,2} - \underline{5}\,R\right)$$

$$+\chi_4\left(4\,\underline{3\underbrace{d^4 + u^4} + e^4}\,\overline{\varphi + \varepsilon}^{\,4} + \frac{2}{3}\overline{d}^{\,2} - \frac{3}{\underline{4}}\,C\bowtie C\right)$$

$$+4\varkappa\,\underline{3\underbrace{d^2 + u^2} + e^2}\,\overline{\eth\,(\varphi + \varepsilon)}^{\,2} - \frac{2}{3}\varkappa R\,\overline{\varphi + \varepsilon}^{\,2}\,\underline{3\underbrace{d^2 + u^2} + e^2}$$