$$\underbrace{\eth\, d + \begin{array}{c|c}\omega_3 & 0\\ \hline 0 & \widetilde{\omega}_3\end{array}} = d\, {\begin{array}{c|c}\omega_3 & 0\\ \hline 0 & \widetilde{\omega}^2_3\end{array}}^{\perp} + {\begin{array}{c|c}\omega^2_3 & 0\\ \hline 0 & \widetilde{\omega}^2_3\end{array}}^{\perp} =$$

$$\begin{array}{c|c|c}
\begin{matrix} d\omega ⧗ \begin{array}{c|c}1 & 0\\ \hline 0 & 1\end{array} + \\ \dfrac{\underbrace{\psi+\varepsilon^*}\underbrace{\varphi+\varepsilon}-I}{\varkappa} ⧗ \begin{array}{c|c} d^* d - r/s & 0 \\ \hline 0 & e^* e - r/s\end{array}\end{matrix} & \varkappa \underbrace{\psi+\varepsilon^*}\underbrace{\varphi+\varepsilon} ⧗ \begin{bmatrix} d^* u \\ 0\end{bmatrix} & -\varkappa\Gamma \underbrace{\psi+\varepsilon^*} \sqcap ⧗ \begin{array}{c|c} d^* & 0 \\ \hline 0 & e^*\end{array} \\ \hline
\varkappa \underbrace{\psi+\varepsilon^*}\underbrace{\varphi+\varepsilon} ⧗ \begin{bmatrix} u^* d & 0\end{bmatrix} & \begin{matrix} d\bar{\omega} ⧗ 1 + \\ \dfrac{\underbrace{\psi+\varepsilon^*}\underbrace{\varphi+\varepsilon}-I}{\varkappa} ⧗ \underbrace{u^* u - r/s}\end{matrix} & -\varkappa\Gamma \underbrace{\psi+\varepsilon^*} \sqcap ⧗ \begin{bmatrix} u^* & 0\end{bmatrix} \\ \hline
\sqcap\, (\varphi+\varepsilon)\, \Gamma ⧗ \begin{array}{c|c} d & 0 \\ \hline 0 & e\end{array} & \sqcap \underbrace{\varphi+\varepsilon}\Gamma ⧗ \begin{bmatrix} u \\ 0\end{bmatrix} & \begin{matrix} d\Omega ⧗ \begin{array}{c|c}1 & 0\\ \hline 0 & 1\end{array} + \\ \dfrac{\underbrace{\varphi+\varepsilon}\underbrace{\psi+\varepsilon^*}+\underbrace{\varphi+\varepsilon}\underbrace{\psi+\varepsilon^*}-I}{2\varkappa} ⧗ \begin{array}{c|c} dd^* + uu^* - r/t & 0 \\ \hline 0 & ee^* - r/t\end{array}\end{matrix}
\end{array}$$

$$\begin{array}{c|c|c}
\begin{matrix} d\bar{\omega}_a ⧗ \begin{array}{c|c}\bar{a} & 0\\ \hline 0 & 0\end{array} + \\ \underbrace{d\bar{\omega} - \varkappa r/s \overbrace{\underbrace{\psi+\varepsilon^*}\underbrace{\varphi+\varepsilon} - I}} ⧗ \begin{array}{c|c}0 & 0\\ \hline 0 & 1\end{array}\end{matrix} & 0 & 0 \\ \hline
0 & d\bar{\omega}_a ⧗ \bar{a} & 0 \\ \hline
0 & 0 & \begin{matrix} d\bar{\omega}_a ⧗ \begin{array}{c|c}\bar{a} & 0\\ \hline 0 & 0\end{array} + \\ \underbrace{d\bar{\omega} - \varkappa r/s \overbrace{\underbrace{\psi+\varepsilon^*}\underbrace{\varphi+\varepsilon} - I}} ⧗ \begin{array}{c|c}0 & 0\\ \hline 0 & 1\end{array}\end{matrix}
\end{array}$$

$$d\, {\begin{array}{c|c}\omega_3 & 0\\ \hline 0 & \widetilde{\omega}_3\end{array}}^{\perp} =$$

$$\begin{array}{c|c|c}
\begin{matrix} d\omega ⧗ \begin{array}{c|c}1 & 0\\ \hline 0 & 1\end{array} + \\ \varkappa \underbrace{\varepsilon^*\varphi+\psi\varepsilon} ⧗ \begin{array}{c|c} d^* d - r/s & 0 \\ \hline 0 & e^* e - r/s\end{array}\end{matrix} & \varkappa \underbrace{\varepsilon^*\varphi+\psi\varepsilon} ⧗ \begin{bmatrix} d^* u \\ 0\end{bmatrix} & \varkappa\Gamma \underbrace{\varepsilon^*\Omega - \omega\varepsilon^* - d\psi} ⧗ \begin{array}{c|c} d^* & 0 \\ \hline 0 & e^*\end{array} \\ \hline
\varkappa\, (\varepsilon^*\varphi+\psi\varepsilon) ⧗ \begin{bmatrix} u^* d & 0\end{bmatrix} & \begin{matrix} d\bar{\omega} ⧗ 1 + \\ \varkappa \underbrace{\varepsilon^*\varphi+\psi\varepsilon} ⧗ \underbrace{u^* u - r/s}\end{matrix} & \varkappa\Gamma \underbrace{\varepsilon^*\Omega - \bar{\omega}\varepsilon^* - d\psi} ⧗ \begin{bmatrix} u^* & 0\end{bmatrix} \\ \hline
\underbrace{d\varphi + \Omega\varepsilon - \varepsilon\omega}\Gamma ⧗ \begin{array}{c|c} d & 0 \\ \hline 0 & e\end{array} & \underbrace{d\varphi + \Omega\varepsilon - \varepsilon\bar{\omega}}\Gamma ⧗ \begin{bmatrix} u \\ 0\end{bmatrix} & \begin{matrix} d\Omega ⧗ \begin{array}{c|c}1 & 0\\ \hline 0 & 1\end{array} + \\ \varkappa \dfrac{\varepsilon\psi + \varphi\varepsilon^* + \varepsilon\psi + \varphi\varepsilon^*}{2} ⧗ \begin{array}{c|c} dd^* + uu^* - r/t & 0 \\ \hline 0 & ee^* - r/t\end{array}\end{matrix}
\end{array}$$

$$
\begin{array}{c|c|c}
\begin{array}{c} d\bar{\omega}_a \mathbin{⧗} \begin{array}{c|c} \bar{a} & 0 \\ \hline 0 & 0 \end{array} + \\ \overbrace{d\bar{\omega} - \varkappa r/s\,\underbrace{\varepsilon^*\varphi + \psi\varepsilon}} \mathbin{⧗} \begin{array}{c|c} 0 & 0 \\ \hline 0 & 1 \end{array} \end{array} & 0 & 0 \\
\hline
0 & d\bar{\omega}_a \mathbin{⧗} \bar{a} & 0 \\
\hline
0 & 0 & \begin{array}{c} d\bar{\omega}_a \mathbin{⧗} \begin{array}{c|c} \bar{a} & 0 \\ \hline 0 & 0 \end{array} + \\ \overbrace{d\bar{\omega} - \varkappa r/s\,\underbrace{\varepsilon^*\varphi + \psi\varepsilon}} \mathbin{⧗} \begin{array}{c|c} 0 & 0 \\ \hline 0 & 1 \end{array} \end{array}
\end{array}
$$

$$
\begin{array}{c|c} \omega_3^2 & 0 \\ \hline 0 & \widetilde{\omega}_3 \end{array}^{\perp} =
$$

$$
\begin{array}{c|c|c}
\varkappa\psi\varphi \mathbin{⧗} \begin{array}{c|c} d^*d - r/s & 0 \\ \hline 0 & e^*e - r/s \end{array} & \varkappa\psi\varphi \mathbin{⧗} \begin{bmatrix} d^*u \\ 0 \end{bmatrix} & \varkappa\Gamma\,\underbrace{\psi\Omega - \omega\psi} \mathbin{⧗} \begin{array}{c|c} d^* & 0 \\ \hline 0 & e^* \end{array} \\
\hline
\varkappa\psi\varphi \mathbin{⧗} \begin{bmatrix} u^*d & 0 \end{bmatrix} & \varkappa\psi\varphi \mathbin{⧗} \underbrace{u^*u - r/s} & \varkappa\Gamma\,\underbrace{\psi\Omega - \bar{\omega}\psi} \mathbin{⧗} \begin{bmatrix} u^* & 0 \end{bmatrix} \\
\hline
\underbrace{\Omega\varphi - \varphi\omega}\Gamma \mathbin{⧗} \begin{array}{c|c} d & 0 \\ \hline 0 & e \end{array} & \underbrace{\Omega\varphi - \varphi\bar{\omega}}\Gamma \mathbin{⧗} \begin{bmatrix} u \\ 0 \end{bmatrix} & \begin{array}{c} \Omega \times \Omega \mathbin{⧗} \begin{array}{c|c} 1 & 0 \\ \hline 0 & 1 \end{array} + \\ \varkappa \dfrac{\varphi\psi + \varphi\psi}{2} \mathbin{⧗} \begin{array}{c|c} dd^* + uu^* - r/t & 0 \\ \hline 0 & ee^* - r/t \end{array} \end{array}
\end{array}
$$

$$
\begin{array}{c|c|c}
\begin{array}{c} \bar{\omega}_b^0 \times \bar{\omega}_c^1 \mathbin{⧗} \begin{array}{c|c} \bar{b}\bar{c} & 0 \\ \hline 0 & 0 \end{array} \\ -\varkappa r/s\psi^0\varphi^1 \mathbin{⧗} \begin{array}{c|c} 0 & 0 \\ \hline 0 & 1 \end{array} \end{array} & 0 & 0 \\
\hline
0 & \bar{\omega}_b^0 \times \bar{\omega}_c^1 \mathbin{⧗} \bar{b}\bar{c} & 0 \\
\hline
0 & 0 & \begin{array}{c} \bar{\omega}_b^0 \times \bar{\omega}_c^1 \mathbin{⧗} \begin{array}{c|c} \bar{b}\bar{c} & 0 \\ \hline 0 & 0 \end{array} \\ -\varkappa r/s\psi^0\varphi^1 \mathbin{⧗} \begin{array}{c|c} 0 & 0 \\ \hline 0 & 1 \end{array} \end{array}
\end{array}
$$

$$
\pitchfork\varphi = d\varphi + \Omega\varphi - \varphi\omega \quad \varphi\pitchfork = d\varphi + \omega\varphi - \varphi\Omega: \quad \pitchfork\Omega = d\Omega + \Omega \times \Omega \quad \pitchfork\left(\omega_a \mathbin{⧗} a\right) = d\omega_a \mathbin{⧗} a + \omega_b \times \omega_c \mathbin{⧗} bc
$$

$$
\varepsilon^*\varepsilon = I = \varepsilon^*\varepsilon: \quad \varepsilon^*\varepsilon = 0 = \varepsilon^*\varepsilon: \quad \varepsilon\varepsilon^* + \varepsilon^*\varepsilon = I
$$

$$
\pitchfork\underbrace{\varphi + \varepsilon} = d\underbrace{\varphi + \varepsilon} + \Omega\underbrace{\varphi + \varepsilon} - \underbrace{\varphi + \varepsilon}\omega = d\varphi + \Omega\varphi + \Omega\varepsilon - \varphi\omega - \varepsilon\omega
$$

$$
\underbrace{\psi + \varepsilon^*}\pitchfork = d\underbrace{\psi + \varepsilon^*} + \omega\underbrace{\psi + \varepsilon^*} - \underbrace{\psi + \varepsilon^*}\Omega = d\psi + \omega\psi + \omega\varepsilon^* - \psi\Omega - \varepsilon^*\Omega
$$

$$
\pitchfork\underbrace{\varphi + \varepsilon} = d\underbrace{\varphi + \varepsilon} + \Omega\underbrace{\varphi + \varepsilon} - \underbrace{\varphi + \varepsilon}\bar{\omega} = d\varphi + \Omega\varphi + \Omega\varepsilon - \varphi\bar{\omega} - \varepsilon\bar{\omega}
$$

$$
\underbrace{\psi + \varepsilon^*}\pitchfork = d\underbrace{\psi + \varepsilon^*} + \bar{\omega}\,\underbrace{\psi + \varepsilon^*} - \underbrace{\psi + \varepsilon^*}\Omega = d\psi + \bar{\omega}\,\psi + \bar{\omega}\,\varepsilon^* - \psi\Omega - \varepsilon^*\Omega
$$

$$\psi\varphi = \langle \psi\varphi \rangle$$

$$\left\| \eth\, d + \begin{array}{c|c} \omega_3 & 0 \\ \hline 0 & \widetilde{\omega}_3 \end{array} \right\|^2 = s\left\| d\omega \right\|^2 + t\left\| d\Omega \right\|^2 + 4\langle \widetilde{x} \rangle \left\| d\left(\omega_a \mathbin{\rtimes} a\right) \right\|^2$$

$$+ \left(\left\langle \overbrace{d^* d}^2 x + \overbrace{u^* u}^2 x + \overbrace{e^* e}^2 y \right\rangle - r^2/s \right) \left\| \underbrace{\varepsilon^* + \psi}\,\underbrace{\varphi + \varepsilon} - I \right\|^2$$

$$-r\varkappa \left(\left\| \lrcorner\, \underbrace{\varphi + \varepsilon} \right\|^2 + \left\| \underbrace{\varepsilon^* + \psi}\, \lrcorner \right\|^2 \right) + 2\langle u^* d d^* u x \rangle \left\| \underbrace{\varepsilon^* + \psi}\,\underbrace{\varphi + \varepsilon} \right\|^2$$

$$+ \left(\left\langle \overbrace{d^* d}^2 x + \overbrace{u^* u}^2 x + \overbrace{e^* e}^2 y \right\rangle + 2\langle u^* d d^* u x \rangle - r^2/t \right) \left\| \underbrace{\varphi + \varepsilon}\,\underbrace{\varepsilon^* + \psi} + \underbrace{\varphi + \varepsilon}\,\underbrace{\varepsilon^* + \psi} - I \right\|^2 /4$$

$$\text{LHS} = s\left\| d\omega - \varkappa r/s\, \underbrace{\varepsilon^* + \psi}\,\underbrace{\varphi + \varepsilon} - I \right\|^2 + 4\langle \widetilde{x} \rangle \left\| d\underbrace{\omega_a \rtimes a} \right\|^2 + t\left\| d\Omega - \varkappa \frac{r}{2t} \overbrace{\underbrace{\varphi + \varepsilon}\,\underbrace{\varepsilon^* + \psi} + \underbrace{\varphi + \varepsilon}\,\underbrace{\varepsilon^* + \psi} - I} \right\|^2$$

$$+2\langle u^* d d^* u x \rangle \underbrace{\left\| \underbrace{\varepsilon^* + \psi}\,\underbrace{\varphi + \varepsilon} \right\|^2 + \left\| \underbrace{\varphi + \varepsilon}\,\underbrace{\varepsilon^* + \psi} + \underbrace{\varphi + \varepsilon}\,\underbrace{\varepsilon^* + \psi} - I \right\|^2 /4}$$

$$+\varkappa r \left(2\Re \overbrace{\underbrace{\varepsilon^* + \psi}\,\underbrace{\varphi + \varepsilon} - I} \rtimes \underbrace{d\omega - \varkappa r/s \overbrace{\underbrace{\varepsilon^* + \psi}\,\underbrace{\varphi + \varepsilon} - I}} - \left\| \lrcorner\, \underbrace{\varphi + \varepsilon} \right\|^2 - \left\| \underbrace{\varepsilon^* + \psi}\, \lrcorner \right\|^2 + 2\Re \left(d\Omega - \varkappa \frac{r}{2t} \overbrace{\underbrace{\varphi + \varepsilon}\,\underbrace{\varepsilon^* + \psi} \ldots} \right. \right.$$

$$+ \left\langle \overbrace{d^* d}^2 x + \overbrace{u^* u}^2 x + \overbrace{e^* e}^2 y \right\rangle \underbrace{\left\| \underbrace{\varepsilon^* + \psi}\,\underbrace{\varphi + \varepsilon} - I \right\|^2 + \left\| \underbrace{\varphi + \varepsilon}\,\underbrace{\varepsilon^* + \psi} + \underbrace{\varphi + \varepsilon}\,\underbrace{\varepsilon^* + \psi} - I \right\|^2 /4}$$

$$= s\left\| d\omega \right\|^2 + t\left\| d\Omega \right\|^2 + 4\langle \widetilde{x} \rangle \underbrace{\left\| d\underbrace{\omega_a \rtimes a} \right\|^2 - r\varkappa \left\| \lrcorner\, \underbrace{\varphi + \varepsilon} \right\|^2 + \left\| \underbrace{\varepsilon^* + \psi}\, \lrcorner \right\|^2}$$

$$+ \left\| \underbrace{\varepsilon^* + \psi}\,\underbrace{\varphi + \varepsilon} - I \right\|^2 \underbrace{s\frac{r^2}{s^2} - 2\frac{r^2}{s} - \left\langle \overbrace{d^* d}^2 x + \overbrace{u^* u}^2 x + \overbrace{e^* e}^2 y \right\rangle}$$

$$+ \left\| \underbrace{\varphi + \varepsilon}\,\underbrace{\varepsilon^* + \psi} + \underbrace{\varphi + \varepsilon}\,\underbrace{\varepsilon^* + \psi} - I \right\|^2 \left(\frac{t}{4}\frac{r^2}{t^2} + \frac{1}{2}\langle u^* d d^* u x \rangle - \frac{r}{2} r/t + \frac{1}{4} \left\langle \overbrace{d^* d}^2 x + \overbrace{u^* u}^2 x + \overbrace{e^* e}^2 y \right\rangle \right)$$

$$+ \left\| \underbrace{\varepsilon^* + \psi}\,\underbrace{\varphi + \varepsilon} \right\|^2 2\langle u^* d d^* u x \rangle$$

$$\underbrace{\eth\, d + \begin{array}{c|c} \omega_3 & 0 \\ \hline 0 & \widetilde{\omega}_3 \end{array}} =$$

$d\omega \bowtie \begin{array}{c\|c} 1 & 0 \\ \hline 0 & 1 \end{array} +$ $\dfrac{\overbrace{\varphi+\varepsilon}^{2}-1}{\varkappa} \bowtie \begin{array}{c\|c} d^{*}d - r/s & 0 \\ \hline 0 & e^{*}e - r/s \end{array}$	0	$-\varkappa\Gamma(\pitchfork(\varphi+\varepsilon))^{*} \bowtie \begin{array}{c\|c} d^{*} & 0 \\ \hline 0 & e^{*} \end{array}$
0	$-d\omega_{+} \bowtie$ $\dfrac{\overbrace{\varphi+\varepsilon}^{2}-1}{\varkappa} \bowtie (u^{*}u - r/s)$	$-\varkappa\Gamma\overbrace{\pitchfork\underbrace{\varphi+\varepsilon}}^{*} \bowtie \begin{bmatrix} u^{*} & 0 \end{bmatrix}$
$\pitchfork\underbrace{\varphi+\varepsilon}\Gamma \bowtie \begin{array}{c\|c} d & 0 \\ \hline 0 & e \end{array}$	$\pitchfork\underbrace{\varphi+\varepsilon}\Gamma \bowtie \begin{bmatrix} u \\ 0 \end{bmatrix}$	$d\Omega \bowtie \begin{array}{c\|c} 1 & 0 \\ \hline 0 & 1 \end{array} +$ $\dfrac{\overbrace{\varphi+\varepsilon}^{2}-1}{2\varkappa} \bowtie \begin{array}{c\|c} dd^{*} + uu^{*} - r/t & 0 \\ \hline 0 & ee^{*} - r/t \end{array}$

$d\bar{\omega}_{a} \bowtie \begin{array}{c\|c} \bar{a} & 0 \\ \hline 0 & 0 \end{array}$ $+d\omega \bowtie \begin{array}{c\|c} 1/3 & 0 \\ \hline 0 & -1 \end{array} -$ $\varkappa r/s\underbrace{\overbrace{\varphi+\varepsilon}^{2}-1} \bowtie \begin{array}{c\|c} 0 & 0 \\ \hline 0 & 1 \end{array}$	0	0
0	$d\bar{\omega}_{a} \bowtie \bar{a}$ $+d\omega \bowtie 1/3$	0
0	0	$d\bar{\omega}_{a} \bowtie \begin{array}{c\|c} \bar{a} & 0 \\ \hline 0 & 0 \end{array}$ $+d\omega \bowtie \begin{array}{c\|c} 1/3 & 0 \\ \hline 0 & -1 \end{array} -$ $\varkappa r/s\underbrace{\overbrace{\varphi+\varepsilon}^{2}-1} \bowtie \begin{array}{c\|c} 0 & 0 \\ \hline 0 & 1 \end{array}$

$$\phi = \varphi + \varepsilon \Longrightarrow \phi^{*}\phi = 0 = \phi^{*}\phi\phi^{*}\phi = \overbrace{\varphi}^{2} = \phi^{*}\phi: \quad \varphi\varphi^{*} + \varphi\varphi^{*} = \phi^{*}\phi$$

$$\overbrace{\pitchfork\underbrace{\varphi+\varepsilon}}^{*} = \overbrace{d\varphi + \Omega\underbrace{\varphi+\varepsilon} - \underbrace{\varphi+\varepsilon}\omega}^{*} = d\varphi^{*} + \overbrace{\varphi+\varepsilon}^{*}\Omega^{*} - \bar{\omega}\,\overbrace{\varphi+\varepsilon}^{*}$$

$$= d\varphi^{*} - \overbrace{\varphi+\varepsilon}^{*}\Omega + \omega\overbrace{\varphi+\varepsilon}^{*} = \overbrace{\varphi+\varepsilon}^{*}\pitchfork$$

$$d\underbrace{\omega_{a} \bowtie a} = d\underbrace{\omega_{a} \bowtie a - \omega \bowtie 1/3} + \underbrace{\omega_{a} \bowtie a - \omega \bowtie 1/3} \times \underbrace{\omega_{a} \bowtie a - \omega \bowtie 1/3} =$$

$$d\omega_{a} \bowtie a - d\omega \bowtie 1/3 + \omega_{a} \bowtie a \times \omega_{\bowtie} a - \underbrace{\omega_{a} \times \omega + \xi \oplus \omega_{a}} \bowtie a/3 + \xi \oplus \omega \bowtie 1/9 = d\underbrace{\omega_{a} \bowtie a} - d\underbrace{\omega \bowtie 1/3}$$

$$d\underbrace{\bar{\omega}_{a} \bowtie \bar{a}} = d\underbrace{\bar{\omega}_{a} \bowtie \bar{a}} + d\underbrace{\omega \bowtie 1/3}$$

$$\overbrace{\eth\underbrace{d + \begin{array}{c|c} \omega_{3} & 0 \\ \hline 0 & \widetilde{\omega}_{3} \end{array}}}^{2} = P\underbrace{d\omega_{3}^{\mathbb{R}} + \overset{2}{\omega}{}_{3}^{\mathbb{R}}} \ast P\underbrace{d\omega_{3}^{\mathbb{R}} + \overset{2}{\omega}{}_{3}^{\mathbb{R}}} =$$

$$= \|d\omega\|^2 \left(s + 4\langle\tilde{x}/3\rangle\right) + t\,\|d\Omega\|^2 + 4\langle\tilde{x}\rangle \|d\left(\omega_a \wedge a\right)\|^2 + \frac{r}{2} m_H^2 \underbrace{\left\{\overbrace{\|\varphi+\varepsilon\|^2 - 1}^{2} - 4\varkappa \|\lrcorner\,\underbrace{\varphi+\varepsilon}\|^2\right\}}$$

$$\frac{r}{2} m_H^2 = \frac{3}{2}\left\langle \widehat{d^*d}^{\,2} x + \widehat{u^*u}^{\,2} x + \widehat{e^*e}^{\,2} y \right\rangle + \left\langle u^* d d^* u x \right\rangle - \frac{r^2}{s} - \frac{r^2}{2t} =$$

$$\frac{1}{2}\left\langle dd^* + uu^* - r/t\right\rangle^2 x + \frac{1}{2}\left\langle ee^* - r/t\right\rangle^2 y + \left\langle dd^* - r/s\right\rangle^2 x + \left\langle uu^* - r/s\right\rangle^2 x + \left\langle ee^* - r/s\right\rangle^2 y + \frac{3r^2}{s^2}\langle\tilde{y}\rangle$$

$$\text{LHS} = s\,\|d\omega\|^2 + t\,\|d\Omega\|^2 + 4\langle\tilde{x}\rangle \|d\underbrace{\omega_a \wedge a} - d\omega \wedge 1/3\|^2 - r\varkappa \underbrace{\left\{\|\lrcorner\,\underbrace{\varphi+\varepsilon}\|^2 + \|\widehat{\lrcorner\,\underbrace{\varphi+\varepsilon}}^{*}\|^2\right\}}$$

$$+2\left\langle u^* d d^* u x\right\rangle \|\widehat{\varepsilon+\varphi}^{*}\underbrace{\varphi+\varepsilon}\|^2 + \left(\left\langle \widehat{d^*d}^{\,2} x + \widehat{u^*u}^{\,2} x + \widehat{e^*e}^{\,2} y\right\rangle - r^2/s\right)\overbrace{\|\varphi+\varepsilon\|^2 - 1}^{2}$$

$$+\left(\left\langle \widehat{d^*d}^{\,2} x + \widehat{u^*u}^{\,2} x + \widehat{e^*e}^{\,2} y\right\rangle + 2\left\langle u^* d d^* u x\right\rangle - r^2/t\right)\|\|\varphi+\varepsilon\|^2 - I\|^2/4$$

$$= \|d\omega\|^2 \left(s + 4\langle\tilde{x}/3\rangle\right) + t\,\|d\Omega\|^2 + 4\langle\tilde{x}\rangle\|d\underbrace{\omega_a \wedge a}\|^2 - 2r\varkappa\|\lrcorner\,\underbrace{\varphi+\varepsilon}\|^2$$

$$+\overbrace{\|\varphi+\varepsilon\|^2 - 1}^{2}\left(\frac{3}{2}\left\langle \widehat{d^*d}^{\,2} x + \widehat{u^*u}^{\,2} x + \widehat{e^*e}^{\,2} y\right\rangle + \left\langle u^* d d^* u x\right\rangle - \frac{r^2}{s} - \frac{r^2}{2t}\right)$$

$$\Leftarrow \|d\underbrace{\omega_a \wedge a} - d\omega \wedge 1/3\|^2 = \|d\underbrace{\omega_a \wedge a}\|^2 + 3\|d\omega\|^2/9$$

$$\lambda \wedge \lambda = 2\lambda^2$$

$$\text{RHS} = \frac{1}{2}\left\langle\widehat{dd^*}^{\,2} x\right\rangle + \frac{1}{2}\left\langle\widehat{uu^*}^{\,2} x\right\rangle + \frac{r^2}{2t^2}\langle x\rangle + \left\langle dd^* uu^* x\right\rangle - r/t\left\langle d^*dx + u^*ux\right\rangle$$

$$+\frac{1}{2}\left\langle\widehat{ee^*}^{\,2} y\right\rangle + \frac{r^2}{2t^2}\langle y\rangle - r/t\left\langle ee^* y\right\rangle + \left\langle\widehat{dd^*}^{\,2} x\right\rangle + \frac{r^2}{s^2}\langle x\rangle - 2r/s\left\langle dd^* x\right\rangle$$

$$+\left\langle\widehat{uu^*}^{\,2} x\right\rangle + \frac{r^2}{s^2}\langle x\rangle - 2r/s\left\langle uu^* x\right\rangle + \left\langle\widehat{ee^*}^{\,2} y\right\rangle + \frac{r^2}{s^2}\langle x\rangle - 2r/s\left\langle ee^* y\right\rangle + \frac{3r^2}{s^2}\langle\tilde{y}\rangle$$

$$= \frac{3}{2}\left\langle\widehat{dd^*}^{\,2} x + \widehat{uu^*}^{\,2} x + \widehat{ee^*}^{\,2} y\right\rangle + \left\langle dd^* uu^* x\right\rangle$$

$$+\frac{r^2}{s^2}\left\langle 2x + y + 3\tilde{y}\right\rangle + \frac{r^2}{2t^2}\left\langle x + y\right\rangle - \underbrace{r/t + \frac{2r}{s}}\left\langle\widehat{dd^*}^{\,2} x + \widehat{uu^*}^{\,2} x + \widehat{ee^*}^{\,2} y\right\rangle$$

$$= \frac{3}{2}\left\langle\widehat{dd^*}^{\,2} x + \widehat{uu^*}^{\,2} x + \widehat{ee^*}^{\,2} y\right\rangle + \left\langle dd^* uu^* x\right\rangle - \frac{r^2}{s} - \frac{r^2}{2t} = \text{MHS}$$

$$\Longleftarrow \frac{r^2}{s^2}\llcorner 2x+y+3\tilde{y}\lrcorner + \frac{r^2}{2t^2}\llcorner x+y\lrcorner - \underbrace{r/t+\frac{2r}{s}}\llcorner \overbrace{d^*d}^{2} x + \overbrace{u^*u}^{2} x + \overbrace{e^*e}^{2} y\lrcorner = \frac{r^2}{s^2}s + \frac{r^2}{2t^2}t - \underbrace{r/t+\frac{2r}{s}} r = -\frac{r^2}{s} - \frac{r^2}{2t}$$

$$\lceil \eth\,\underbrace{d + \begin{array}{c|c} \omega_3 & 0 \\ \hline 0 & \tilde{\omega}_3 \end{array}} \rceil^{2} = P\,\underbrace{d\omega_3^{\mathbb{R}} + \overset{2}{\omega}{}_3^{\mathbb{R}}} \,\bar{\times}\, P\,\underbrace{d\omega_3^{\mathbb{R}} + \overset{2}{\omega}{}_3^{\mathbb{R}}} =$$

$$= \lceil d\omega \rceil^{2} \left(s + 4\,\llcorner \tilde{x}/3 \lrcorner\right) + t\,\lceil d\Omega \rceil^{2} + 4\,\llcorner \tilde{x} \lrcorner \lceil d\,\underbrace{\omega_a \,\underline{\bar{\times}}\, a} \rceil^{2} + \frac{r}{2} m_H^2 \underbrace{\overbrace{\lceil \varphi+\varepsilon \rceil^{2} - 1}^{2} - 4\varkappa \lceil \lrcorner\,\underbrace{\varphi+\varepsilon} \rceil^{2}}$$

$$\frac{r}{2} m_H^2 = \frac{3}{2}\llcorner \overbrace{d^*d}^{2} x + \overbrace{u^*u}^{2} x + \overbrace{e^*e}^{2} y \lrcorner + \llcorner u^* d d^* u x \lrcorner - \frac{r^2}{s} - \frac{r^2}{2t}$$

$$\text{LHS} = s\,\underbrace{d\omega - \varkappa r/s\,\underbrace{\lceil \varphi+\varepsilon \rceil^{2}} - 1} \,\bar{\times}\, \overbrace{d\omega - \varkappa r/s\,\underbrace{\lceil \varphi+\varepsilon \rceil^{2} - 1}} + t\,\overbrace{d\Omega - \varkappa \frac{r}{2t}\underbrace{\lceil \varphi+\varepsilon \rceil^{2} - 1}} \,\bar{\times}\, \overbrace{d\Omega - \varkappa \frac{r}{2t}\underbrace{\lceil \varphi+\varepsilon \rceil^{2} - 1}}$$

$$+4\,\llcorner \tilde{x} \lrcorner\,\underbrace{d\omega_a \,\underline{\bar{\times}}\, a + d\omega \,\underline{\bar{\times}}\, 1/3} \,\bar{\times}\, \underbrace{d\omega_a \,\underline{\bar{\times}}\, a + d\omega \,\underline{\bar{\times}}\, 1/3}$$

$$+r\varkappa \left(\frac{2}{\overbrace{d\omega - \varkappa r/s\,\underbrace{\lceil \varphi+\varepsilon \rceil^{2}} - 1}} \,\bar{\times}\, \overbrace{\lceil \varphi+\varepsilon \rceil^{2} - 1} + 2\,\underbrace{d\Omega - \varkappa \frac{r}{2t}\underbrace{\lceil \varphi+\varepsilon \rceil^{2} - 1}} \,\bar{\times}\, \frac{1}{2}\underbrace{\lceil \varphi+\varepsilon \rceil^{2} - 1} - \lrcorner\,\underbrace{\varphi+\varepsilon} \,\bar{\times}\, \lrcorner\,\underbrace{\varphi+\varepsilon} - \lrcorner\,\overbrace{\varphi+\varepsilon}^{*} \,\bar{\times}\, \lrcorner\,\overbrace{\varphi \right.$$

$$+\llcorner \overbrace{d^*d}^{2} x + \overbrace{u^*u}^{2} x + \overbrace{e^*e}^{2} y \lrcorner \underbrace{\lceil \varphi+\varepsilon \rceil^{2} - 1} \,\bar{\times}\, \underbrace{\lceil \varphi+\varepsilon \rceil^{2} - 1} + \frac{1}{4}\underbrace{\lceil \varphi+\varepsilon \rceil^{2} - 1} \,\bar{\times}\, \underbrace{\lceil \varphi+\varepsilon \rceil^{2} - 1} + 2\,\llcorner u^* d d^* u x \lrcorner \frac{1}{4}\underbrace{\lceil \varphi+\varepsilon \rceil^{2} - 1} \,\bar{\times}\, \underbrace{\lceil \varphi+\varepsilon}$$

$$\varphi \,\bar{\times}\, \varphi = 2\varphi \,\bar{\times}\, \varphi\,\llcorner \frac{1}{3}\frac{1}{3} \lrcorner = 3\frac{1}{9} = \frac{1}{3}$$