

$$F \in \mathbb{R}^{n \times n} \quad h \in \mathbb{R}^n \quad \tilde{F} = F + h \tilde{h}$$

$$\begin{cases} {}_i \bar{\underline{\text{Pf}}}^j = \bar{\underline{\text{Pf}}}^j \in \mathbb{R} \\ {}_i \underline{\lambda} \bar{\underline{\text{Pf}}} = \lambda \bar{\underline{\text{Pf}}}^j \end{cases}$$

$$\begin{cases} {}_i\bar{\Gamma}^{\bar{\text{H}}\text{H}}\Gamma^j = \bar{\Gamma}_i^{\bar{\text{H}}\text{H}}\Gamma^j \in {}^{\bar{\text{H}}}\Delta_{\infty}^{\bar{\text{H}}} \Delta \mathbb{R} \\ {}_i\bar{\Gamma}_{\lambda}^{\lambda}\bar{\Gamma}^j = \lambda\bar{\Gamma}_i^{\bar{\text{H}}\text{H}}\Gamma^j = \lambda\bar{\Gamma}_i^j\Gamma^j \quad i\bar{\Gamma}_k^j|\bar{\text{H}}\text{H}\Gamma^j = k\bar{\Gamma}_i^j\Gamma^j \end{cases}$$

$$\left\{ \begin{array}{l} {}_{\mu}\underline{\lambda}\bar{\underline{\text{Pf}}}\underline{\gamma}^{\nu} = \overline{\underline{\lambda}\underline{\text{Pf}}\underline{\gamma}}^{\nu} \in {}^{\hbar}\underline{\Delta}_{\infty}^{\hbar}\Delta \mathbb{R} \\ {}_{\mu}\underline{\lambda}\underline{\lambda}\bar{\underline{\text{Pf}}} = \underline{\lambda}\underline{\lambda}\bar{\underline{\text{Pf}}}\underline{\gamma}^{\nu}\underline{\lambda} = \overline{\underline{\lambda}\underline{\mu}\underline{\text{Pf}}\underline{\gamma}}^{\nu}\underline{\lambda} \quad {}_{\mu}\underline{\text{Pf}} = \overline{\underline{\text{Pf}}\underline{\gamma}}^{\nu}\underline{\lambda} \end{array} \right.$$

$$d\underset{ij}{\underline{\mathbf{H}}} = \underset{i}{\overline{\mathbf{H}}}\underset{j}{\underline{\mathbf{H}}} \underset{kj}{\underline{\mathbf{H}}^k} + \underset{j}{\overline{\mathbf{H}}}\underset{i}{\underline{\mathbf{H}}} \underset{ki}{\underline{\mathbf{H}}^k} \text{ metric}$$

$$\begin{aligned}
& \lambda \underset{-}{\underline{\mathbf{L}}} | d\underset{ij}{\underline{\mathbf{H}}} = d\underset{\lambda \underset{-}{\underline{\mathbf{L}}}}{\underline{\mathbf{L}}} \underset{i}{\underline{\mathbf{L}}} \times \underset{j}{\underline{\mathbf{L}}} = d\underset{\lambda \underset{-}{\underline{\mathbf{L}}}}{\underline{\mathbf{L}}} \underset{i}{\underline{\mathbf{L}}} \times \underset{j}{\underline{\mathbf{L}}} + \underset{i}{\underline{\mathbf{L}}} \times d\underset{\lambda \underset{-}{\underline{\mathbf{L}}}}{\underline{\mathbf{L}}} \underset{j}{\underline{\mathbf{L}}} = \\
& \underset{i}{\overline{\mathbf{H}}} \underset{-\lambda}{\underline{\mathbf{H}}^k} \underset{k}{\underline{\mathbf{L}}} \times \underset{j}{\underline{\mathbf{L}}} + \underset{i}{\underline{\mathbf{L}}} \times \underset{j}{\overline{\mathbf{H}}} \underset{-\lambda}{\underline{\mathbf{H}}^k} \underset{k}{\underline{\mathbf{L}}} = \underset{i}{\overline{\mathbf{H}}} \underset{-\lambda}{\underline{\mathbf{H}}^k} \underset{kj}{\underline{\mathbf{H}}} + \underset{ik}{\underline{\mathbf{H}}} \underset{j}{\overline{\mathbf{H}}} \underset{-\lambda}{\underline{\mathbf{H}}^k} = \\
& \underbrace{\lambda \underset{-}{\underline{\mathbf{L}}} | \underset{i}{\overline{\mathbf{H}}} \underset{-\lambda}{\underline{\mathbf{H}}^k} \underset{kj}{\underline{\mathbf{H}}}}_{ik} + \underset{ik}{\underline{\mathbf{H}}} \underbrace{\lambda \underset{-}{\underline{\mathbf{L}}} | \underset{j}{\overline{\mathbf{H}}} \underset{-\lambda}{\underline{\mathbf{H}}^k}}_{jk} \\
& d\underset{-}{\underline{\mathbf{H}}}^{mn} = -\underset{-}{\underline{\mathbf{H}}}^{mi} \underbrace{d\underset{ij}{\underline{\mathbf{H}}}}_{ij} \underset{-}{\underline{\mathbf{H}}}^{jn} = -\underset{-}{\underline{\mathbf{H}}}^{mk} \underset{k}{\overline{\mathbf{H}}} \underset{-}{\underline{\mathbf{H}}}^n - \underset{-}{\underline{\mathbf{H}}}^{nk} \underset{k}{\overline{\mathbf{H}}} \underset{-}{\underline{\mathbf{H}}}^m \\
& 0 = d\underset{ij}{\underline{\mathbf{H}}} \underset{-}{\underline{\mathbf{H}}}^{jn} = \underbrace{d\underset{ij}{\underline{\mathbf{H}}}}_{ij} \underset{-}{\underline{\mathbf{H}}}^{jn} + \underset{ij}{\underline{\mathbf{H}}} d\underset{-}{\underline{\mathbf{H}}}^{jn} \Rightarrow \\
& d\underset{-}{\underline{\mathbf{H}}}^{mn} = \underset{-}{\underline{\mathbf{H}}}^{mi} \underset{ij}{\underline{\mathbf{H}}} \underbrace{d\underset{-}{\underline{\mathbf{H}}}^{jn}}_{ij} = -\underset{-}{\underline{\mathbf{H}}}^{mi} \underbrace{d\underset{ij}{\underline{\mathbf{H}}}}_{ij} \underset{-}{\underline{\mathbf{H}}}^{jn} = \\
& -\underset{i}{\underline{\mathbf{H}}}^{mi} \underset{-}{\underline{\mathbf{H}}} \underset{kj}{\underline{\mathbf{H}}^k} \underset{kj}{\underline{\mathbf{H}}} \underset{-}{\underline{\mathbf{H}}}^{jn} - \underset{-}{\underline{\mathbf{H}}}^{mi} \underset{ik}{\underline{\mathbf{H}}} \underset{j}{\overline{\mathbf{H}}} \underset{-}{\underline{\mathbf{H}}^k} \underset{-}{\underline{\mathbf{H}}}^{jn} = -\underset{i}{\underline{\mathbf{H}}}^{mi} \underset{-}{\underline{\mathbf{H}}} \underset{i}{\overline{\mathbf{H}}} \underset{-}{\underline{\mathbf{H}}}^n - \underset{j}{\overline{\mathbf{H}}} \underset{-}{\underline{\mathbf{H}}}^m \underset{-}{\underline{\mathbf{H}}}^{jn}
\end{aligned}$$