

$$\overline{\overline{A}} = d(\overline{A})$$

$$B \times \left(\overline{\overline{A}} \right)^m = B \times \overline{\overline{A}}^m - B \times \overline{\overline{A}}^m - \underbrace{B \times B \times \overline{\overline{A}}^m}_{+ \underbrace{B \overline{\overline{A}}^k}_{k} \underbrace{\overline{\overline{A}}^k} - \underbrace{B \overline{\overline{A}}^k}_{k} \underbrace{\overline{\overline{A}}^k}}$$

$$\begin{aligned} LHS &= \overline{\overline{A}}^m = \left(\overline{\overline{A}}_k - \overline{\overline{A}}_{k+1} + \dots - \overline{\overline{A}}_m \right) A^m \\ &= \underbrace{\overline{\overline{A}}_k - \overline{\overline{A}}_k}_{B \times \overline{\overline{A}}_k} - \underbrace{\overline{\overline{A}}_k - \overline{\overline{A}}_k}_{B \times \overline{\overline{A}}_k} - \underbrace{B \times \overline{\overline{A}}_k}_{- B \times \overline{\overline{A}}_k} = \\ &\quad \underbrace{B \overline{\overline{A}}_k - B \overline{\overline{A}}_k}_{B \overline{\overline{A}}_k} - \underbrace{B \times B \overline{\overline{A}}_k}_{B \times \overline{\overline{A}}_k} = RHS \end{aligned}$$

$$\begin{aligned} \overline{\overline{A}}_{ij}^m &= \overline{\overline{A}}_i \overline{\overline{A}}_j \overline{\overline{A}}_{\ell}^m = \overline{\overline{A}}_i \times \overline{\overline{A}}_{\ell j}^m - \overline{\overline{A}}_j \times \overline{\overline{A}}_{\ell i}^m \\ &- \left(\overline{\overline{A}}_{ij}^k - \overline{\overline{A}}_{ji}^k \right) \overline{\overline{A}}_{\ell k}^m + \overline{\overline{A}}_{\ell i}^k \overline{\overline{A}}_{kj}^m - \overline{\overline{A}}_{\ell j}^k \overline{\overline{A}}_{ki}^m \\ &\quad \overline{\overline{A}}_i \times \overline{\overline{A}}_j = \overline{\overline{A}}_i \overline{\overline{A}}_j - \overline{\overline{A}}_j \overline{\overline{A}}_i = \left(\overline{\overline{A}}_{ij}^k - \overline{\overline{A}}_{ji}^k \right) \overline{\overline{A}}_k \\ &\quad \underline{\underline{1}}_{\lambda} \overline{\overline{A}}_{\mu\nu} \underline{\underline{\gamma}}^{\lambda} = \underline{\underline{1}}_{\mu} \underline{\underline{1}}_{\nu} \overline{\overline{A}}_{\lambda}^{\lambda} \\ &\quad = \underline{\underline{1}}_{\mu} \times \underline{\underline{1}}_{\lambda\nu} \overline{\overline{A}}_{\lambda}^{\lambda} - \underline{\underline{1}}_{\nu} \times \underline{\underline{1}}_{\lambda\mu} \overline{\overline{A}}_{\lambda}^{\lambda} \\ &\quad + \left(\overline{\overline{A}}_{\mu\nu} \right)_{\mu\nu}^{\lambda} - \overline{\overline{A}}_{\lambda\nu}^{\lambda} \overline{\overline{A}}_{\mu\lambda}^{\lambda} \\ &\quad \overline{\overline{A}}_{ij}^m = \overline{\overline{A}}_i \overline{\overline{A}}_j \overline{\overline{A}}_{\ell}^m = \overline{\overline{A}}_i \times \overline{\overline{A}}_{\ell j}^m - \overline{\overline{A}}_j \times \overline{\overline{A}}_{\ell i}^m \\ &- \overline{\overline{A}}_{ij}^k \overline{\overline{A}}_{\ell k}^m + \overline{\overline{A}}_{ji}^k \overline{\overline{A}}_{\ell k}^m + \overline{\overline{A}}_{\ell i}^k \overline{\overline{A}}_{kj}^m - \overline{\overline{A}}_{\ell j}^k \overline{\overline{A}}_{ki}^m \\ &\quad \underline{\underline{1}}_{\mu} \times \underline{\underline{1}}_{\nu} = 0 \end{aligned}$$

$$\begin{aligned}
& \overline{\mathcal{A} \bar{\mathcal{H}} \mathcal{H}}_{ij} = \overline{\mathcal{A} \bar{\mathcal{H}} \mathcal{H} \mathcal{A}}_{ij} = {}_i \mathcal{A} \star {}_j \overline{\mathcal{A} \bar{\mathcal{H}}} \mathcal{H} \mathcal{A} - {}_j \mathcal{A} \star {}_i \overline{\mathcal{A} \bar{\mathcal{H}}} \mathcal{H} \mathcal{A} \\
& - \left({}_{ij} \overline{\mathcal{A} \bar{\mathcal{H}}} \mathcal{H} \mathcal{A}^k - {}_{ji} \overline{\mathcal{A} \bar{\mathcal{H}}} \mathcal{H} \mathcal{A}^k \right) {}_k \overline{\mathcal{A} \bar{\mathcal{H}}} \mathcal{H} \mathcal{A} + {}_{ij} \overline{\mathcal{A} \bar{\mathcal{H}}} \mathcal{H} \mathcal{A} \star {}_j \overline{\mathcal{A} \bar{\mathcal{H}}} \mathcal{H} \mathcal{A} \\
& \underline{\mathcal{A} \bar{\mathcal{H}} \mathcal{H}}_{\mu\nu} = \overline{\underline{\mathcal{A} \bar{\mathcal{H}} \mathcal{H}}}_{\mu\nu} \\
& = {}_{\mu} \mathcal{A} \star {}_{\nu} \overline{\underline{\mathcal{A} \bar{\mathcal{H}}}} \mathcal{H} \mathcal{A} - {}_{\nu} \mathcal{A} \star {}_{\mu} \overline{\underline{\mathcal{A} \bar{\mathcal{H}}}} \mathcal{H} \mathcal{A} + {}_{\mu} \overline{\underline{\mathcal{A} \bar{\mathcal{H}}}} \mathcal{H} \mathcal{A} \star {}_{\nu} \overline{\underline{\mathcal{A} \bar{\mathcal{H}}}} \mathcal{H} \mathcal{A} \\
& \overline{\mathcal{T} \bar{\mathcal{H}} \mathcal{H}}_{\mu\nu} = \overline{\mathcal{T} \bar{\mathcal{H}} \mathcal{H} \mathcal{T}}_{\mu\nu} \\
& = {}_{\mu} \mathcal{A} \star {}_{\nu} \overline{\mathcal{T} \bar{\mathcal{H}} \mathcal{H} \mathcal{T}} - {}_{\nu} \mathcal{A} \star {}_{\mu} \overline{\mathcal{T} \bar{\mathcal{H}} \mathcal{H} \mathcal{T}} + {}_{\mu} \overline{\mathcal{T} \bar{\mathcal{H}} \mathcal{H} \mathcal{T}} \star {}_{\nu} \overline{\mathcal{T} \bar{\mathcal{H}} \mathcal{H} \mathcal{T}} \\
& \overline{\mathcal{T} \bar{\mathcal{H}} \mathcal{H} \mathcal{T}}_{ij} \in \mathbb{R}^{p:q}
\end{aligned}$$

$$k \overline{\mathcal{T} \bar{\mathcal{H}} \mathcal{H} \mathcal{T}}_{ij}^{\ell} \eta + {}_{k\ell} \eta \overline{\mathcal{T} \bar{\mathcal{H}} \mathcal{H} \mathcal{T}}_{ij}^{\ell} = 0$$

$$\begin{aligned}
\text{LHS} &= k \overline{\mathcal{T} \bar{\mathcal{H}} \mathcal{H} \mathcal{T}}_{ij}^{\ell} \mathcal{T} \star {}_m \mathcal{T} + {}_k \mathcal{T} \star {}_l \mathcal{T} {}_m \overline{\mathcal{T} \bar{\mathcal{H}} \mathcal{H} \mathcal{T}}_{ij}^{\ell} \\
&= \overbrace{k \overline{\mathcal{T} \bar{\mathcal{H}} \mathcal{H} \mathcal{T}}_{ij}^{\ell} \mathcal{T} \star {}_m \mathcal{T}} + \overbrace{{}_k \mathcal{T} \star {}_m \overline{\mathcal{T} \bar{\mathcal{H}} \mathcal{H} \mathcal{T}}_{ij}^{\ell}} = \underbrace{{}_k \mathcal{T} \star {}_l \mathcal{T} | \overline{\mathcal{H} \mathcal{H}}_{ij}^{\ell}} + \underbrace{{}_k \mathcal{T} \star {}_m \mathcal{T} | \overline{\mathcal{H} \mathcal{H}}_{ij}^{\ell}} = 0
\end{aligned}$$

$$m \overline{\mathcal{T} \bar{\mathcal{H}} \mathcal{H} \mathcal{T}}_{ij}^{\ell} \eta = {}_i \overline{\mathcal{T} \bar{\mathcal{H}} \mathcal{H} \mathcal{T}}_{mn}^{\ell} \eta$$

$$\begin{aligned}
\text{LHS} &= \overbrace{{}_m \mathcal{T} \star {}_i \mathcal{T} | \overline{\mathcal{H} \mathcal{H}}^{\ell}} \mathcal{T} \star {}_n \mathcal{T} = \overbrace{{}_m \mathcal{T} \star {}_i \mathcal{T} | \overline{\mathcal{H} \mathcal{H}}^{\ell}} \mathcal{T} \star {}_n \mathcal{T} \\
&= \overbrace{{}_m \mathcal{T} \star {}_i \mathcal{T} | \overline{\mathcal{H} \mathcal{H}}^{\ell}} = \overbrace{{}_i \mathcal{T} \star {}_m \mathcal{T} | \overline{\mathcal{H} \mathcal{H}}^{\ell}} = \overbrace{{}_i \mathcal{T} \star {}_m \mathcal{T} | \overline{\mathcal{H} \mathcal{H}}^{\ell}} \mathcal{T} \star {}_j \mathcal{T} = \overbrace{{}_i \mathcal{T} \star {}_m \mathcal{T} | \overline{\mathcal{H} \mathcal{H}}^{\ell}} \mathcal{T} \star {}_j \mathcal{T} = \text{RHS}
\end{aligned}$$

$$\lambda^{\mu\nu} \overline{\underline{\mathcal{U}} \bar{\mathcal{H}} \mathcal{H} \underline{\mathcal{V}}} + \lambda^{\nu\lambda} \overline{\underline{\mathcal{U}} \bar{\mathcal{H}} \mathcal{H} \underline{\mathcal{V}}} + \lambda^{\mu\lambda} \overline{\underline{\mathcal{U}} \bar{\mathcal{H}} \mathcal{H} \underline{\mathcal{V}}} = 0 = \overline{i \bar{\mathcal{U}} \bar{\mathcal{H}} \mathcal{H} \underline{\mathcal{L}}}^{jk} + \overline{j \bar{\mathcal{U}} \bar{\mathcal{H}} \mathcal{H} \underline{\mathcal{L}}}^{ki} + \overline{k \bar{\mathcal{U}} \bar{\mathcal{H}} \mathcal{H} \underline{\mathcal{L}}}^{ij}$$

$$\begin{aligned} \text{LHS}^\times &= \underbrace{\underline{\mathcal{U}} \bar{\mathcal{H}} \mathcal{H} |_{\mu} \underline{\mathcal{U}} :_{\nu} \underline{\mathcal{V}}^\times}_{\lambda} + \underbrace{\underline{\mathcal{U}} \bar{\mathcal{H}} \mathcal{H} |_{\nu} \underline{\mathcal{U}} :_{\lambda} \underline{\mathcal{V}}^\times}_{\mu} + \underbrace{\underline{\mathcal{U}} \bar{\mathcal{H}} \mathcal{H} |_{\lambda} \underline{\mathcal{U}} :_{\mu} \underline{\mathcal{V}}^\times}_{\nu} \\ &= \underbrace{\underline{\mathcal{U}} \underline{\mathcal{U}} :_{\mu} \underline{\mathcal{V}}^\times | \bar{\mathcal{H}} \mathcal{H}}_{\lambda} + \underbrace{\underline{\mathcal{U}} \underline{\mathcal{U}} :_{\nu} \underline{\mathcal{V}}^\times | \bar{\mathcal{H}} \mathcal{H}}_{\mu} + \underbrace{\underline{\mathcal{U}} \bar{\mathcal{H}} \mathcal{H} |_{\lambda} \underline{\mathcal{U}} :_{\mu} \underline{\mathcal{V}}^\times}_{\nu} = 0 \end{aligned}$$

$$\begin{aligned} \text{RHS}^\ell &= \underbrace{i \bar{\mathcal{U}} \underline{\mathcal{U}} :_j \bar{\mathcal{U}} | \bar{\mathcal{H}} \mathcal{H} \underline{\mathcal{L}}^\ell}_{k} + \underbrace{j \bar{\mathcal{U}} \underline{\mathcal{U}} :_k \bar{\mathcal{U}} | \bar{\mathcal{H}} \mathcal{H} \underline{\mathcal{L}}^\ell}_{i} + \underbrace{k \bar{\mathcal{U}} \underline{\mathcal{U}} :_i \bar{\mathcal{U}} | \bar{\mathcal{H}} \mathcal{H} \underline{\mathcal{L}}^\ell}_{j} \\ &= \underbrace{\bar{\mathcal{U}} \bar{\mathcal{H}} \mathcal{H} |_j \bar{\mathcal{U}} :_k \bar{\mathcal{U}}}_{i} + \underbrace{\bar{\mathcal{U}} \bar{\mathcal{H}} \mathcal{H} |_k \bar{\mathcal{U}} :_i \bar{\mathcal{U}}}_{j} + \underbrace{\bar{\mathcal{U}} \bar{\mathcal{H}} \mathcal{H} |_i \bar{\mathcal{U}} :_j \bar{\mathcal{U}}}_{k} = 0 \end{aligned}$$