

$$2 \, \mathfrak{U}_{k\ell} \, \mathfrak{U}_{ij} \, \mathfrak{H}^\ell = \mathfrak{U}_{i-} \, \mathfrak{U}_{kj} + \mathfrak{U}_{j-} \, \mathfrak{U}_{ki} - \mathfrak{U}_{k-} \, \mathfrak{U}_{ij}$$

$$\mathfrak{U}_{ij} = \varrho^2 \, \mathfrak{U}_{ij} \quad \text{neue metric}$$

$$\bar{\mathcal{H}}_i \mathcal{H}_j^\ell - \bar{\mathcal{H}}_j \mathcal{H}_i^\ell = \overline{\mathcal{A}_i}^\ell \delta_j + \overline{\mathcal{A}_j}^\ell \delta_i - \partial_i^k \mathcal{A}_{jk}$$

$$\begin{aligned}
& 2 \varrho^2 \overline{\eta_{kl}}_i \bar{\eta}_j^\ell - \overline{\eta_{kl}}_i \bar{\eta}_j^\ell = 2 \overline{\eta}_{kl}{}_i \bar{\eta}_j^\ell - 2 \varrho^2 \overline{\eta}_{kl}{}_i \bar{\eta}_j^\ell \\
&= \underline{1}_{kj} \overline{\eta}_{kl}{}_i + \underline{1}_{ki} \overline{\eta}_{kl}{}_j - \underline{1}_{ij} \overline{\eta}_{kl}{}_k - \varrho^2 \overline{\eta_{kl}}_{ij} + \overline{\eta_{kl}}_{ji} - \overline{\eta_{kl}}_{ik} \\
&= \underline{1}_{kj} \overline{\varrho^2 \eta}_{kl} + \underline{1}_{ki} \overline{\varrho^2 \eta}_{kl} - \underline{1}_{ij} \overline{\varrho^2 \eta}_{kl} - \varrho^2 \overline{\eta_{kl}}_{ij} + \overline{\eta_{kl}}_{ji} - \overline{\eta_{kl}}_{ik} \\
&= \overline{\underline{1}_{kj} \varrho^2} \overline{\eta}_{kl}{}_i + \overline{\underline{1}_{ki} \varrho^2} \overline{\eta}_{kl}{}_j - \overline{\underline{1}_{ij} \varrho^2} \overline{\eta}_{kl}{}_k = 2 \varrho \overline{\underline{1}_{kj} \varrho} \overline{\eta}_{kl}{}_i + \overline{\underline{1}_{ki} \varrho} \overline{\eta}_{kl}{}_j - \overline{\underline{1}_{ij} \varrho} \overline{\eta}_{kl}{}_k \\
&\Rightarrow \overline{\eta_{kl}}_i \bar{\eta}_j^\ell - \overline{\eta_{kl}}_i \bar{\eta}_j^\ell = \overline{\eta}_{kl}{}_i \bar{\eta}_j^\ell \left(\overline{\underline{1}_{kj} \varrho} + \overline{\underline{1}_{ki} \varrho} - \overline{\underline{1}_{ij} \varrho} \right) = \overline{\underline{1}_{kj} \varrho} \overline{\eta}_{kl}{}_i \bar{\eta}_j^\ell + \overline{\underline{1}_{ki} \varrho} \overline{\eta}_{kl}{}_j \bar{\eta}_i^\ell - \overline{\underline{1}_{ij} \varrho} \overline{\eta}_{kl}{}_k \bar{\eta}_i^\ell = \overline{\underline{1}_{kj} \varrho}^\ell \delta_j + \overline{\underline{1}_{ki} \varrho}^\ell \delta_i - \overline{\underline{1}_{ij} \varrho}^\ell \delta_k
\end{aligned}$$

$$2 \, \mathfrak{H}_{k\ell} \, \mathfrak{H}_{ij}^{\ell} = \mathfrak{F}_{ij} \, \mathfrak{H}_{kj} + \mathfrak{F}_{ij} \, \mathfrak{H}_{ki} - \mathfrak{F}_{ij} \, \mathfrak{H}_{ij}$$

$$\mathfrak{H}_{ij} = \varrho^2 \, \mathfrak{H}_{ij} \quad \text{neue metric}$$

$$\bar{\mathfrak{H}}_i \mathfrak{H}_j^\ell - \bar{\mathfrak{H}}_j \mathfrak{H}_i^\ell = \overline{\mathfrak{F}_0}^\ell \delta_j + \overline{\mathfrak{F}_0}^\ell \delta_i - \partial_\ell^k \mathfrak{H}_{ij}$$

$$\begin{aligned}
2\varrho^2 \overline{\eta_{kl} \eta_j^\ell} - \overline{\eta_{kl} \eta_j^\ell} &= 2\eta_{kl} \bar{\eta}_j^\ell - 2\varrho^2 \overline{\eta_{kl} \eta_j^\ell} \\
&= \overline{\eta_{kj}} + \overline{\eta_{ki}} - \overline{\eta_{ij}} - \varrho^2 \overline{\eta_{kj} + \eta_{ki} - \eta_{ij}} \\
&= \overline{\varrho^2 \eta_{kj}} + \overline{\varrho^2 \eta_{ki}} - \overline{\varrho^2 \eta_{ij}} - \varrho^2 \overline{\eta_{kj} + \eta_{ki} - \eta_{ij}} \\
&= \overline{\varrho^2 \eta_{kj}} + \overline{\varrho^2 \eta_{ki}} - \overline{\varrho^2 \eta_{ij}} = 2\varrho \overline{\varrho \eta_{kj}} + \overline{\varrho \eta_{ki}} - \overline{\varrho \eta_{ij}} \\
\Rightarrow \overline{\eta_j^\ell} - \overline{\eta_j^\ell} &= \overline{\eta_j^\ell} \left(\overline{\varrho \eta_{kj}} + \overline{\varrho \eta_{ki}} - \overline{\varrho \eta_{ij}} \right) = \overline{\varrho \eta_j^\ell} \overline{\eta_{kj}} + \overline{\varrho \eta_j^\ell} \overline{\eta_{ki}} - \overline{\varrho \eta_j^\ell} \overline{\eta_{ij}} = \overline{\varrho \eta_j^\ell} \delta_j + \overline{\varrho \eta_j^\ell} \delta_i - \overline{\varrho \eta_j^\ell} \delta_i
\end{aligned}$$

liftup tangent metric cisformations

$$2\mathcal{A}_{k\ell} \bar{\mathcal{A}}_{ij} \mathcal{H}^\ell = {}_i\partial_{kj} \mathcal{A} + {}_j\partial_{ki} \mathcal{A} - {}_k\partial_{ij} \mathcal{A}$$

$$\mathcal{A}_{ij} = \varrho^2 \mathcal{A}_{ij}$$

$$\bar{\mathcal{A}}_{ij} \mathcal{H}^\ell - \bar{\mathcal{A}}_{ij} \mathcal{H}^\ell = \widehat{{}_i\partial_{\varrho}^\ell \delta_j} + \widehat{{}_j\partial_{\varrho}^\ell \delta_i} - \widehat{\partial_{ij}^k \mathcal{A}}$$

$$2\varrho^2 \mathcal{A}_{k\ell} \bar{\mathcal{A}}_{ij} \mathcal{H}^\ell - \bar{\mathcal{A}}_{ij} \mathcal{H}^\ell = 2\mathcal{A}_{k\ell} \bar{\mathcal{A}}_{ij} \mathcal{H}^\ell - 2\varrho^2 \mathcal{A}_{k\ell} \bar{\mathcal{A}}_{ij} \mathcal{H}^\ell$$

$$= {}_i\partial_{kj} \mathcal{A} + {}_j\partial_{ki} \mathcal{A} - {}_k\partial_{ij} \mathcal{A} - \varrho^2 \widehat{{}_i\partial_{kj} \mathcal{A} + {}_j\partial_{ki} \mathcal{A} - {}_k\partial_{ij} \mathcal{A}}$$

$$= {}_i\partial \widehat{\varrho^2 \mathcal{A}_{kj}} + {}_j\partial \widehat{\varrho^2 \mathcal{A}_{ki}} - {}_k\partial \widehat{\varrho^2 \mathcal{A}_{ij}} - \varrho^2 \widehat{{}_i\partial_{kj} \mathcal{A} + {}_j\partial_{ki} \mathcal{A} - {}_k\partial_{ij} \mathcal{A}}$$

$$= \widehat{{}_i\partial \varrho^2 \mathcal{A}_{kj}} + \widehat{{}_j\partial \varrho^2 \mathcal{A}_{ki}} - \widehat{{}_k\partial \varrho^2 \mathcal{A}_{ij}} = 2\varrho \widehat{{}_i\partial_{\varrho} \mathcal{A}_{kj}} + \widehat{{}_j\partial_{\varrho} \mathcal{A}_{ki}} - \widehat{{}_k\partial_{\varrho} \mathcal{A}_{ij}}$$

$$\Rightarrow \bar{\mathcal{A}}_{ij} \mathcal{H}^\ell - \bar{\mathcal{A}}_{ij} \mathcal{H}^\ell = \mathcal{H}^{\ell k} \widehat{{}_i\partial_{\varrho} \mathcal{A}_{kj} + {}_j\partial_{\varrho} \mathcal{A}_{ki} - {}_k\partial_{\varrho} \mathcal{A}_{ij}} = \widehat{{}_i\partial_{\varrho} \mathcal{H}^{\ell k} \mathcal{A}_{kj}} + \widehat{{}_j\partial_{\varrho} \mathcal{H}^{\ell k} \mathcal{A}_{ki}} - \mathcal{H}^{\ell k} \widehat{{}_k\partial_{\varrho} \mathcal{A}_{ij}} = \widehat{{}_i\partial_{\varrho}^\ell \delta_j} + \widehat{{}_j\partial_{\varrho}^\ell \delta_i} - \widehat{\partial_{ij}^k \mathcal{A}}$$

$$2\mathcal{A}_{k\ell} \bar{\mathcal{A}}_{ij} \mathcal{H}^\ell = {}_{i-}\mathcal{A}_{kj} + {}_{j-}\mathcal{A}_{ki} - {}_{k-}\mathcal{A}_{ij}$$

$$\mathcal{A}_{ij} = \varrho^2 \mathcal{A}_{ij}$$

$$\bar{\mathcal{A}}_{ij} \mathcal{H}^\ell - \bar{\mathcal{A}}_{ij} \mathcal{H}^\ell = \widehat{{}_{i-}\mathcal{A}_{kj}} + \widehat{{}_{j-}\mathcal{A}_{ki}} - \widehat{{}_{k-}\mathcal{A}_{ij}}$$

$$2\varrho^2 \mathcal{A}_{k\ell} \bar{\mathcal{A}}_{ij} \mathcal{H}^\ell - \bar{\mathcal{A}}_{ij} \mathcal{H}^\ell = 2\mathcal{A}_{k\ell} \bar{\mathcal{A}}_{ij} \mathcal{H}^\ell - 2\varrho^2 \mathcal{A}_{k\ell} \bar{\mathcal{A}}_{ij} \mathcal{H}^\ell$$

$$= {}_{i-}\mathcal{A}_{kj} + {}_{j-}\mathcal{A}_{ki} - {}_{k-}\mathcal{A}_{ij} - \varrho^2 \widehat{{}_{i-}\mathcal{A}_{kj} + {}_{j-}\mathcal{A}_{ki} - {}_{k-}\mathcal{A}_{ij}}$$

$$= {}_{i-}\partial \widehat{\varrho^2 \mathcal{A}_{kj}} + {}_{j-}\partial \widehat{\varrho^2 \mathcal{A}_{ki}} - {}_{k-}\partial \widehat{\varrho^2 \mathcal{A}_{ij}} - \varrho^2 \widehat{{}_{i-}\mathcal{A}_{kj} + {}_{j-}\mathcal{A}_{ki} - {}_{k-}\mathcal{A}_{ij}}$$

$$= \widehat{{}_{i-}\partial \varrho^2 \mathcal{A}_{kj}} + \widehat{{}_{j-}\partial \varrho^2 \mathcal{A}_{ki}} - \widehat{{}_{k-}\partial \varrho^2 \mathcal{A}_{ij}} = 2\varrho \widehat{{}_{i-}\partial_{\varrho} \mathcal{A}_{kj}} + \widehat{{}_{j-}\partial_{\varrho} \mathcal{A}_{ki}} - \widehat{{}_{k-}\partial_{\varrho} \mathcal{A}_{ij}}$$

$$\Rightarrow \bar{\mathcal{A}}_{ij} \mathcal{H}^\ell - \bar{\mathcal{A}}_{ij} \mathcal{H}^\ell = \mathcal{H}^{\ell k} \widehat{{}_{i-}\partial_{\varrho} \mathcal{A}_{kj} + {}_{j-}\partial_{\varrho} \mathcal{A}_{ki} - {}_{k-}\partial_{\varrho} \mathcal{A}_{ij}} = \widehat{{}_{i-}\partial_{\varrho} \mathcal{H}^{\ell k} \mathcal{A}_{kj}} + \widehat{{}_{j-}\partial_{\varrho} \mathcal{H}^{\ell k} \mathcal{A}_{ki}} - \mathcal{H}^{\ell k} \widehat{{}_{k-}\partial_{\varrho} \mathcal{A}_{ij}} = \widehat{{}_{i-}\partial_{\varrho}^\ell \delta_j} + \widehat{{}_{j-}\partial_{\varrho}^\ell \delta_i} - \widehat{{}_{k-}\partial_{\varrho}^k \mathcal{A}_{ij}}$$