

$$d\mathfrak{h} = \bar{\mathfrak{h}}$$

$$\begin{cases} \mathfrak{A}_{ij} = {}_i\mathfrak{b}_j \mathfrak{b}\mathfrak{A} = {}_i\mathfrak{b}_j \mathfrak{b}\mathfrak{A} \\ \mathfrak{A}_{\mu\nu} = {}_{\mu}\mathfrak{b}_{\mu} {}_{\nu}\mathfrak{b}\mathfrak{A} = {}_{\mu}\mathfrak{b}_{\nu} {}_{\nu}\mathfrak{b}\mathfrak{A} \\ {}_{mn}\eta = \mathfrak{A}_{mn} = {}_m\mathfrak{r}_n \mathfrak{r}\mathfrak{A} = {}_m\mathfrak{r}_n \mathfrak{r}\mathfrak{A} \text{ cst} \end{cases} \quad \begin{matrix} {}_m\bar{\mathfrak{A}}\mathfrak{h}^n = \left(\bar{\mathfrak{A}}\mathfrak{h}^n\right)_\ell^m \\ {}_m\bar{\mathfrak{A}}\mathfrak{h}^n = \left(\bar{\mathfrak{A}}\mathfrak{h}^n\right)_m^n \\ {}_m\bar{\mathfrak{A}}\mathfrak{h}^n = \left(\bar{\mathfrak{A}}\mathfrak{h}^n\right)_{\ell m}^m \end{matrix}$$

$$2\underbrace{{}_m\bar{\mathfrak{A}}\mathfrak{h}^n}_{nk} \mathfrak{A} = {}_\ell\mathfrak{b}| \bar{\mathfrak{A}}| {}_{mk} + {}_m\mathfrak{b}| \bar{\mathfrak{A}}| {}_{\ell k} - {}_k\mathfrak{b}| \bar{\mathfrak{A}}| {}_{\ell m} + \underbrace{{}_\ell\mathfrak{b}_m\mathfrak{b}|\bar{\mathfrak{A}}^n}_{nk} \mathfrak{A} - \underbrace{{}_k\mathfrak{b}_m\mathfrak{b}|\bar{\mathfrak{A}}^n}_{n\ell} \mathfrak{A} - \underbrace{{}_k\mathfrak{b}_\ell\mathfrak{b}|\bar{\mathfrak{A}}^n}_{nm} \mathfrak{A}$$

$$\underbrace{{}_m\bar{\mathfrak{A}}\mathfrak{h}^n}_{nk} \mathfrak{A} = \alpha \underbrace{{}_\ell\mathfrak{b}|\bar{\mathfrak{A}}}_{mk} + \beta \underbrace{{}_m\mathfrak{b}|\bar{\mathfrak{A}}}_{\ell k} + \gamma \underbrace{{}_k\mathfrak{b}|\bar{\mathfrak{A}}}_{\ell m} + \lambda \underbrace{{}_\ell\mathfrak{b}_m\mathfrak{b}|\bar{\mathfrak{A}}^n}_{nk} \mathfrak{A} + \mu \underbrace{{}_k\mathfrak{b}_m\mathfrak{b}|\bar{\mathfrak{A}}^n}_{n\ell} \mathfrak{A} + \nu \underbrace{{}_k\mathfrak{b}_\ell\mathfrak{b}|\bar{\mathfrak{A}}^n}_{nm} \mathfrak{A}$$

$$2\underbrace{{}_m\bar{\mathfrak{A}}\mathfrak{h}^n}_{nk} \mathfrak{A} = {}_{\ell-}\mathfrak{b}| \bar{\mathfrak{A}}| {}_{mk} + {}_{m-}\mathfrak{b}| \bar{\mathfrak{A}}| {}_{\ell k} - {}_{k-}\mathfrak{b}| \bar{\mathfrak{A}}| {}_{\ell m} + \underbrace{{}_{\ell-}{}_{m-}\mathfrak{b}|\overline{d\mathcal{V}}^n}_{nk} \mathfrak{A} - \underbrace{{}_{k-}{}_{m-}\mathfrak{b}|\overline{d\mathcal{V}}^n}_{n\ell} \mathfrak{A} - \underbrace{{}_{k-}{}_{\ell-}\mathfrak{b}|\overline{d\mathcal{V}}^n}_{nm} \mathfrak{A}$$

$$= {}_{\ell-}\mathfrak{b}| \bar{\mathfrak{A}}| {}_{mk} + {}_{m-}\mathfrak{b}| \bar{\mathfrak{A}}| {}_{\ell k} - {}_{k-}\mathfrak{b}| \bar{\mathfrak{A}}| {}_{\ell m}$$

$$2\underbrace{{}_m\bar{\mathfrak{A}}\mathfrak{h}^n}_{nk} \mathfrak{A} = {}_\ell\mathfrak{r}| \bar{\mathfrak{A}}| {}_{mk} + {}_m\mathfrak{r}| \bar{\mathfrak{A}}| {}_{\ell k} - {}_k\mathfrak{r}| \bar{\mathfrak{A}}| {}_{\ell m} + \underbrace{{}_\ell\mathfrak{r}_m\mathfrak{r}|\bar{\mathfrak{A}}^n}_{nk} \mathfrak{A} - \underbrace{{}_k\mathfrak{r}_m\mathfrak{r}|\bar{\mathfrak{A}}^n}_{n\ell} \mathfrak{A} - \underbrace{{}_k\mathfrak{r}_\ell\mathfrak{r}|\bar{\mathfrak{A}}^n}_{nm} \mathfrak{A}$$

$$= \underbrace{{}_\ell\mathfrak{r}_m\mathfrak{r}|\bar{\mathfrak{A}}^n}_{nk} \mathfrak{A} - \underbrace{{}_k\mathfrak{r}_m\mathfrak{r}|\bar{\mathfrak{A}}^n}_{n\ell} \mathfrak{A} - \underbrace{{}_k\mathfrak{r}_\ell\mathfrak{r}|\bar{\mathfrak{A}}^n}_{nm} \mathfrak{A}$$

$$\text{metric} \quad \begin{cases} \underbrace{\bar{\mathbf{A}}\mathbf{h}^m}_{i}{}_{mj} + \underbrace{\bar{\mathbf{A}}\mathbf{h}^m}_{j}{}_{mi} = \bar{\mathbf{A}}_{ij} & \beta = 1/2 : \gamma = -\alpha : \mu = -\lambda \\ \underbrace{\bar{\mathbf{A}}\mathbf{h}^m}_{i}{}_{mj} + \underbrace{\bar{\mathbf{A}}\mathbf{h}^m}_{j}{}_{mi} = \bar{\mathbf{A}}_{ij} & \beta = 1/2 : \gamma = -\alpha \\ \underbrace{\bar{\mathbf{A}}\mathbf{h}^m}_{i}{}_{mj} + \underbrace{\bar{\mathbf{A}}\mathbf{h}^m}_{j}{}_{mi} = \bar{\mathbf{A}}_{ij} & \mu = -\lambda \end{cases}$$

$$\begin{aligned}
& {}_k \mathbf{A} \overbrace{\underbrace{\bar{\mathbf{A}}\mathbf{h}^m}_{i}{}_{mj} + \underbrace{\bar{\mathbf{A}}\mathbf{h}^m}_{j}{}_{mi}} = \underbrace{\bar{\mathbf{A}}\mathbf{h}^m}_{k}{}_{ij} + \underbrace{\bar{\mathbf{A}}\mathbf{h}^m}_{k}{}_{ji} = \alpha {}_i \mathbf{A} | \bar{\mathbf{A}}_{kj} + \beta {}_k \mathbf{A} | \bar{\mathbf{A}}_{ij} + \gamma {}_j \mathbf{A} | \bar{\mathbf{A}}_{ik} \\
& + \lambda \underbrace{\mathbf{A} {}_k \mathbf{A} | \bar{\mathbf{A}}^n}_{nj} \bar{\mathbf{A}} + \mu \underbrace{\mathbf{A} {}_k \mathbf{A} | \bar{\mathbf{A}}^n}_{ni} \bar{\mathbf{A}} + \nu \underbrace{\mathbf{A} {}_i \mathbf{A} | \bar{\mathbf{A}}^n}_{nk} \bar{\mathbf{A}} + \alpha {}_j \mathbf{A} | \bar{\mathbf{A}}_{ki} + \beta {}_k \mathbf{A} | \bar{\mathbf{A}}_{ji} + \gamma {}_i \mathbf{A} | \bar{\mathbf{A}}_{jk} \\
& + \lambda \underbrace{\mathbf{A} {}_k \mathbf{A} | \bar{\mathbf{A}}^n}_{ni} \bar{\mathbf{A}} + \mu \underbrace{\mathbf{A} {}_k \mathbf{A} | \bar{\mathbf{A}}^n}_{nj} \bar{\mathbf{A}} + \nu \underbrace{\mathbf{A} {}_i \mathbf{A} | \bar{\mathbf{A}}^n}_{nk} \bar{\mathbf{A}} = {}_k \mathbf{A} | \bar{\mathbf{A}}_{ij} \\
& {}_k \mathbf{A} \overbrace{\underbrace{\bar{\mathbf{A}}\mathbf{h}^m}_{i}{}_{mj} + \underbrace{\bar{\mathbf{A}}\mathbf{h}^m}_{j}{}_{mi}} = \underbrace{\bar{\mathbf{A}}\mathbf{h}^m}_{k}{}_{mj} + \underbrace{\bar{\mathbf{A}}\mathbf{h}^m}_{k}{}_{mi} = \\
& \alpha {}_i \mathbf{A} | \bar{\mathbf{A}}_{kj} + \beta {}_k \mathbf{A} | \bar{\mathbf{A}}_{ij} + \gamma {}_j \mathbf{A} | \bar{\mathbf{A}}_{ik} + \alpha {}_j \mathbf{A} | \bar{\mathbf{A}}_{ki} + \beta {}_k \mathbf{A} | \bar{\mathbf{A}}_{ji} + \gamma {}_i \mathbf{A} | \bar{\mathbf{A}}_{jk} = {}_k \mathbf{A} | \bar{\mathbf{A}}_{ij} \\
& {}_k \mathbf{A} \overbrace{\underbrace{\bar{\mathbf{A}}\mathbf{h}^m}_{i}{}_{mj} + \underbrace{\bar{\mathbf{A}}\mathbf{h}^m}_{j}{}_{mi}} = \underbrace{\bar{\mathbf{A}}\mathbf{h}^m}_{k}{}_{mj} + \underbrace{\bar{\mathbf{A}}\mathbf{h}^m}_{k}{}_{mi} \\
& = \lambda \underbrace{\mathbf{A} {}_i \mathbf{A} | \bar{\mathbf{A}}^n}_{nj} \bar{\mathbf{A}} + \mu \underbrace{\mathbf{A} {}_i \mathbf{A} | \bar{\mathbf{A}}^n}_{ni} \bar{\mathbf{A}} + \nu \underbrace{\mathbf{A} {}_i \mathbf{A} | \bar{\mathbf{A}}^n}_{nk} \bar{\mathbf{A}} \\
& + \lambda \underbrace{\mathbf{A} {}_j \mathbf{A} | \bar{\mathbf{A}}^n}_{ni} \bar{\mathbf{A}} + \mu \underbrace{\mathbf{A} {}_j \mathbf{A} | \bar{\mathbf{A}}^n}_{nj} \bar{\mathbf{A}} + \nu \underbrace{\mathbf{A} {}_j \mathbf{A} | \bar{\mathbf{A}}^n}_{nk} \bar{\mathbf{A}} = {}_k \mathbf{A} | \bar{\mathbf{A}}_{ij}
\end{aligned}$$

$$\text{tors free } \begin{cases} \bar{\mathbf{L}}^i - \mathbf{A}^j \bar{\mathbf{M}}^i = 0 & \alpha = \beta : \lambda = 1/2 : \mu = \nu \\ \bar{\mathbf{M}}^j \bar{\mathbf{L}}^i = 0 & \alpha = \beta \\ \bar{\mathbf{L}}^i - \mathbf{A}^j \bar{\mathbf{M}}^i = 0 & \lambda = 1/2 : \mu = \nu \end{cases}$$

$$\overbrace{_{p\mathbf{A}_q\mathbf{B}^j\bar{\mathbf{L}}^i\mathbf{M}^j}^{\bar{\mathbf{L}}^i-\mathbf{A}^j\bar{\mathbf{M}}^i}}_{i\ell} = \underbrace{_{p\mathbf{A}\mathbf{B}^j}^{\mathbf{A}\mathbf{B}^j}}_{q\mathbf{B}^i} \underbrace{\mathbf{B}^i}_{j} \underbrace{\bar{\mathbf{M}}^i}_{i\ell} - \underbrace{_{q\mathbf{A}\mathbf{B}^j}^{\mathbf{A}\mathbf{B}^j}}_{p\mathbf{B}^i} \underbrace{\mathbf{B}^i}_{j} \underbrace{\bar{\mathbf{M}}^i}_{i\ell} = \underbrace{_{q\mathbf{A}\mathbf{B}^i}^{\mathbf{A}\mathbf{B}^i}}_{p\mathbf{B}^i} \underbrace{\mathbf{B}^i}_{p} \underbrace{\bar{\mathbf{M}}^i}_{i\ell} - \underbrace{_{p\mathbf{A}\mathbf{B}^i}^{\mathbf{A}\mathbf{B}^i}}_{q\mathbf{B}^i} \underbrace{\mathbf{B}^i}_{p} \underbrace{\bar{\mathbf{M}}^i}_{i\ell}$$

$$= {}_q\bar{\mathbf{L}}^i \mathbf{B}^i_{i\ell} - {}_p\bar{\mathbf{L}}^i \mathbf{B}^i_{i\ell} = \alpha {}_p\mathbf{A}^i \bar{\mathbf{L}}^i_{q\ell} + \beta {}_q\mathbf{A}^i \bar{\mathbf{L}}^i_{p\ell} + \gamma {}_\ell\mathbf{A}^i \bar{\mathbf{L}}^i_{pq} + \lambda {}_p\mathbf{A}_q\mathbf{B}^i\bar{\mathbf{L}}^i_{n\ell} + \mu {}_\ell\mathbf{A}_q\mathbf{B}^i\bar{\mathbf{L}}^i_{np} + \nu {}_\ell\mathbf{A}_p\mathbf{B}^i\bar{\mathbf{L}}^i_{nq}$$

$$- \alpha {}_q\mathbf{A}^i \bar{\mathbf{L}}^i_{p\ell} - \beta {}_p\mathbf{A}^i \bar{\mathbf{L}}^i_{q\ell} - \gamma {}_\ell\mathbf{A}^i \bar{\mathbf{L}}^i_{qp} - \lambda {}_q\mathbf{A}_p\mathbf{B}^i\bar{\mathbf{L}}^i_{n\ell} - \mu {}_\ell\mathbf{A}_p\mathbf{B}^i\bar{\mathbf{L}}^i_{nq} - \nu {}_\ell\mathbf{A}_q\mathbf{B}^i\bar{\mathbf{L}}^i_{np} = {}_p\mathbf{A}_q\mathbf{B}^i\bar{\mathbf{L}}^i_{n\ell}$$

$$\overbrace{_{p\mathbf{U}_q\mathbf{U}_l\mathbf{U}^j\bar{\mathbf{L}}^i}^{\mathbf{U}^j\bar{\mathbf{L}}^i}}_{i\ell} = \underbrace{_{p\mathbf{U}_q\mathbf{U}_l}^{\mathbf{U}_q\mathbf{U}_l}}_{q\mathbf{U}_l} \underbrace{\mathbf{U}_l^i}_{j} \underbrace{\bar{\mathbf{L}}^i}_{i\ell} - \underbrace{_{q\mathbf{U}_q\mathbf{U}_l}^{\mathbf{U}_q\mathbf{U}_l}}_{p\mathbf{U}_l} \underbrace{\mathbf{U}_l^i}_{j} \underbrace{\bar{\mathbf{L}}^i}_{i\ell} = \underbrace{_{q\mathbf{U}_p\mathbf{U}_l}^{\mathbf{U}_p\mathbf{U}_l}}_{p\mathbf{U}_l} \underbrace{\mathbf{U}_l^i}_{i\ell} - \underbrace{_{p\mathbf{U}_q\mathbf{U}_l}^{\mathbf{U}_q\mathbf{U}_l}}_{q\mathbf{U}_l} \underbrace{\mathbf{U}_l^i}_{i\ell}$$

$$= {}_q\bar{\mathbf{L}}^i \mathbf{B}^i_{i\ell} - {}_p\bar{\mathbf{L}}^i \mathbf{B}^i_{i\ell} = \alpha {}_p\mathbf{U}^i \bar{\mathbf{L}}^i_{q\ell} + \beta {}_q\mathbf{U}^i \bar{\mathbf{L}}^i_{p\ell} + \gamma {}_\ell\mathbf{U}^i \bar{\mathbf{L}}^i_{pq} - \alpha {}_q\mathbf{U}^i \bar{\mathbf{L}}^i_{p\ell} - \beta {}_p\mathbf{U}^i \bar{\mathbf{L}}^i_{q\ell} - \gamma {}_\ell\mathbf{U}^i \bar{\mathbf{L}}^i_{qp} = 0$$

$$\overbrace{_{p\mathbf{T}_q\mathbf{T}_l\mathbf{T}^j\bar{\mathbf{L}}^i}^{\mathbf{T}^j\bar{\mathbf{L}}^i}}_{i\ell} = \underbrace{_{p\mathbf{T}_q\mathbf{T}_l}^{\mathbf{T}_q\mathbf{T}_l}}_{q\mathbf{T}_l} \underbrace{\mathbf{T}_l^i}_{j} \underbrace{\bar{\mathbf{L}}^i}_{i\ell} - \underbrace{_{q\mathbf{T}_q\mathbf{T}_l}^{\mathbf{T}_q\mathbf{T}_l}}_{p\mathbf{T}_l} \underbrace{\mathbf{T}_l^i}_{j} \underbrace{\bar{\mathbf{L}}^i}_{i\ell} = \underbrace{_{q\mathbf{T}_p\mathbf{T}_l}^{\mathbf{T}_p\mathbf{T}_l}}_{p\mathbf{T}_l} \underbrace{\mathbf{T}_l^i}_{i\ell} - \underbrace{_{p\mathbf{T}_q\mathbf{T}_l}^{\mathbf{T}_q\mathbf{T}_l}}_{q\mathbf{T}_l} \underbrace{\mathbf{T}_l^i}_{i\ell} = {}_q\bar{\mathbf{L}}^i \mathbf{B}^i_{i\ell} - {}_p\bar{\mathbf{L}}^i \mathbf{B}^i_{i\ell}$$

$$= \lambda \underbrace{_{p\mathbf{T}_q\mathbf{T}_l\mathbf{T}^n}^{\mathbf{T}^n\bar{\mathbf{L}}^n}}_{n\ell} + \mu \underbrace{_{\ell\mathbf{T}_q\mathbf{T}_l\mathbf{T}^n}^{\mathbf{T}^n\bar{\mathbf{L}}^n}}_{np} + \nu \underbrace{_{\ell\mathbf{T}_p\mathbf{T}_l\mathbf{T}^n}^{\mathbf{T}^n\bar{\mathbf{L}}^n}}_{nq} - \lambda \underbrace{_{q\mathbf{T}_p\mathbf{T}_l\mathbf{T}^n}^{\mathbf{T}^n\bar{\mathbf{L}}^n}}_{n\ell} - \mu \underbrace{_{\ell\mathbf{T}_p\mathbf{T}_l\mathbf{T}^n}^{\mathbf{T}^n\bar{\mathbf{L}}^n}}_{nq} - \nu \underbrace{_{\ell\mathbf{T}_q\mathbf{T}_l\mathbf{T}^n}^{\mathbf{T}^n\bar{\mathbf{L}}^n}}_{np} = \underbrace{_{p\mathbf{T}_q\mathbf{T}_l\mathbf{T}^n}^{\mathbf{T}^n\bar{\mathbf{L}}^n}}_{n\ell}$$

$$\mathbf{h} \xrightarrow{\mathbf{L}} {}^{\mathbb{C}}\mathbb{R}^n$$

$$0 = d \underline{m\delta^n} = d \underline{{}_m\nabla^\lambda} \underline{{}_\lambda\mathbf{L}^n} = \underline{{}_m\nabla^\lambda} \underline{{}_\lambda\mathbf{L}^n} + \underline{{}_m\nabla^\lambda} \underline{{}_\lambda\mathbf{L}^n}$$

$${}_m\mathbb{L}^n - {}_m\nabla^\mu {}_\mu \mathbf{L}^\nu {}_\nu \mathbf{L}^n = \underline{{}_m\nabla^\lambda} \underline{{}_\lambda\mathbf{L}^n} = - \underline{{}_m\nabla^\lambda} \underline{{}_\lambda\mathbf{L}^n}$$