$$\prod_{k \geqslant 1} \underbrace{1 + zq^k} = \sum_{n}^{\mathbb{N}} z^n \prod_{1 \leqslant j \leqslant n} \frac{q^j}{1 - q^j}$$

$$\sum_{n}^{\mathbb{N}} {}^{\sharp}\daleth_n \, z^n = {}^{z}\daleth = \prod_{k \geqslant 1} \underbrace{1 + zq^k} = \underbrace{1 + zq} \prod_{k \geqslant 2} \underbrace{1 + zq^k} = \underbrace{1 + zq} \prod_{h \geqslant 1} \underbrace{1 + \overset{\frown}{zq}\, q^h} = \underbrace{1 + zq}\, {}^{zq}\daleth$$

$$= \underbrace{1 + zq} \sum_{n}^{\mathbb{N}} {}^{\sharp}\daleth_n \, \overset{\frown}{zq}^{\,n} = \underbrace{1 + zq} \sum_{n}^{\mathbb{N}} {}^{\sharp}\daleth_n \, q^n \, z^n = \sum_{n}^{\mathbb{N}} {}^{\sharp}\daleth_n \, q^n \, z^n + {}^{\sharp}\daleth_n \, q^{n+1} \, z^{n+1}$$

$$\Longrightarrow {}^{\sharp}\daleth_{n+1} \underbrace{1 - q^{n+1}} = {}^{\sharp}\daleth_n \, q^{n+1} \Vdash {}^{\sharp}\daleth_n = \prod_{1 \leqslant j \leqslant n} \frac{q^j}{1 - q^j}$$

$$\prod_{k \geqslant 1} \frac{1}{1 - zq^k} = \sum_{n}^{\mathbb{N}} z^n \prod_{1 \leqslant j \leqslant n} \frac{q}{1 - q^j}$$

$$\prod_{i}^{\mathbb{N}} \underbrace{1 - q^{2i+2}} \underbrace{1 + zq^{2i+1}} \underbrace{1 + z^{-1} q^{2i+1}} = \sum_{n}^{\mathbb{Z}} z^n \, q^{n^2}$$