$$\mathcal{H}\psi = E\psi$$

$$q \in \mathbb{R}^3 \llcorner 0 \text{ config space}$$

$$q{:}p \in \mathbb{R}^3 \llcorner 0 \times {}_3\mathbb{R} \text{ c-phase space}$$

kinematisch

$$\mathbb{R} \xleftarrow{\xi:\eta:\zeta} \mathbb{R}^3 \llcorner 0 \times {}_3\mathbb{R} \xrightarrow{x:y:z} \mathbb{R}$$

$$\rceil \in {}^{\mathbb{R}^3 \llcorner 0} \underset{m}{\overset{2}{\triangle}} \mathbb{C} \text{ q-Hilbert space}$$

$$\int_{dxdydz}^{\mathbb{R}^3 \llcorner 0} \underbrace{\overline{{}_{x:y:z}\rceil}^{\,2}}_{\text{Aufenthalts-Wahrscheinlichkeit in } x:y:z} = 1$$

dynamisch

$$1/2 = \frac{h}{2\pi i}$$

$$\overline{\xi:\eta:\zeta} = 1/2\, \underbrace{\partial_x^2{:}\partial_y^2{:}\partial_z^2}$$

$${}_{q:p}\mathcal{H} = T - V = \frac{1}{2} m v^2 - \frac{e^2}{r} = \frac{p^2}{2m} - \frac{e^2}{\ulcorner q \urcorner}$$

$$\overline{\mathcal{H}} = \frac{1/2^2}{2m} \underbrace{\partial_x^2 + \partial_y^2 + \partial_z^2} - \frac{e^2}{\left(x^2 + y^2 + z^2\right)^{1/2}}$$

$$= \frac{1/2^2}{2m} \frac{1}{r^2} \left(\partial_r \left(r^2\right) \partial_r + \frac{1}{\sin^2 \vartheta} \left(\partial_\vartheta \left(\sin \vartheta\right) \partial_\vartheta + \frac{1}{\sin^2 \vartheta} \partial_\varphi^2 \right) \right) - \frac{e^2}{r}$$

$$ {}^{q}\eta = \frac{e^{-r/a}}{\pi^{\,1/2} a^{3/2}} = \frac{e^{-\left(x^2+y^2+z^2\right)^{\,1/2}/a}}{\pi^{\,1/2} a^{3/2}} $$

$$ \frac{1}{r^2}\,\partial_r\, r^2\,\partial_r\, e^{-kr} = \frac{1}{r^2}\,\partial_r\, r^2\,\left(-k e^{-kr}\right) $$

$$ = \frac{-k}{r^2}\,\partial_r\,\left(r^2 e^{-kr}\right) = \frac{-k}{r^2}\left(2r e^{-kr} - k r^2 e^{-kr}\right) = e^{-kr}\left(k^2 - \frac{2k}{r}\right) $$

$$ \underbrace{\frac{1/2^2}{2m}\,\frac{1}{r^2}\partial_r r^2 \partial_r - \frac{e^2}{r}}\, e^{-kr} = e^{-kr}\,\frac{1/2^2}{2m}\left(k^2 - \frac{2k}{r}\right) - \frac{e^2}{r}\, e^{-kr} $$

$$ = e^{-kr}\left(\frac{1/2^2 k^2}{2m} - \underbrace{\left(\frac{1/2^2 k}{m} + e^2\right)}_{=\,0}\frac{1}{r}\right) = \frac{1/2^2 k^2}{2m}\, e^{-kr} $$

$$ \partial_x\, e^{-k\left(x^2+y^2+z^2\right)^{\,1/2}} = -k\, e^{-k\left(x^2+y^2+z^2\right)^{\,1/2}}\,\partial_x\left(x^2+y^2+z^2\right)^{1/2} $$

$$ = -k\, e^{-k\left(x^2+y^2+z^2\right)^{\,1/2}}\,\frac{1}{2}\left(x^2+y^2+z^2\right)^{-\,1/2} 2x = -kx\, e^{-k\left(x^2+y^2+z^2\right)^{\,1/2}}\left(x^2+y^2+z^2\right)^{-\,1/2} $$

$$ \partial_x\left(x e^{-k\left(x^2+y^2+z^2\right)^{\,1/2}}\left(x^2+y^2+z^2\right)^{-\,1/2}\right) = e^{-k\left(x^2+y^2+z^2\right)^{\,1/2}}\left(x^2+y^2+z^2\right)^{-\,1/2} $$

$$ +x\,\underbrace{-kx e^{-k\left(x^2+y^2+z^2\right)^{\,1/2}}\left(x^2+y^2+z^2\right)^{-\,1/2}} + x\, e^{-k\left(x^2+y^2+z^2\right)^{\,1/2}}\,\frac{-1}{2}\left(x^2+y^2+z^2\right)^{-3/2} 2x $$

$$ = e^{-k\left(x^2+y^2+z^2\right)^{\,1/2}}\left(x^2+y^2+z^2\right)^{-\,1/2}\left(1 - kx^2 - kx^2\left(x^2+y^2+z^2\right)^{-1}\right) $$

$$ \partial_x^2\, e^{-k\left(x^2+y^2+z^2\right)^{\,1/2}} = e^{-k\left(x^2+y^2+z^2\right)^{\,1/2}}\left(x^2+y^2+z^2\right)^{-\,1/2}\left(-k + k^2 x^2 + k^2 x^2\left(x^2+y^2+z^2\right)^{-1}\right) $$

$$ \underbrace{\partial_x^2 + \partial_y^2 + \partial_z^2}\, e^{-k\left(x^2+y^2+z^2\right)^{\,1/2}} $$

$$ = e^{-k\left(x^2+y^2+z^2\right)^{\,1/2}}\left(x^2+y^2+z^2\right)^{-\,1/2}\left(-3k + k^2\left(x^2+y^2+z^2\right) + k^2\left(x^2+y^2+z^2\right)\left(x^2+y^2+z^2\right)^{-1}\right) $$

$$ = e^{-k\left(x^2+y^2+z^2\right)^{\,1/2}}\left(x^2+y^2+z^2\right)^{-\,1/2}\left(k^2 - 3k + k^2\left(x^2+y^2+z^2\right)\right) $$

$$ \Longrightarrow \left(\frac{1/2^2}{2m}\,\underbrace{\partial_x^2 + \partial_y^2 + \partial_z^2} - \frac{e^2}{\left(x^2+y^2+z^2\right)^{\,1/2}}\right) e^{-k\left(x^2+y^2+z^2\right)^{\,1/2}} $$

$$= \frac{1/2^2}{2m} e^{-k\left(x^2+y^2+z^2\right)^{1/2}} \left(x^2+y^2+z^2\right)^{-1/2} \left(k^2 - 3k + k^2 \left(x^2+y^2+z^2\right)\right)$$

$$- \frac{e^2}{\left(x^2+y^2+z^2\right)^{1/2}} e^{-k\left(x^2+y^2+z^2\right)^{1/2}} =$$

$$\left(k^2 - 3k\right) \frac{1/2^2}{2m} e^{-k\left(x^2+y^2+z^2\right)^{1/2}} \left(x^2+y^2+z^2\right)^{-1/2}$$

$$+ k^2 \frac{1/2^2}{2m} e^{-k\left(x^2+y^2+z^2\right)^{1/2}} \left(x^2+y^2+z^2\right)^{1/2} \left(k^2 \frac{1/2^2}{2m} - e^2\right) = E\, e^{-k\left(x^2+y^2+z^2\right)^{1/2}}$$