

$\mathbb{R}$ :

$$\begin{array}{ccc}
 \begin{array}{c} {}^{\ell}\mathbb{C}_r^{\mathbb{C}} \\ \downarrow \\ {}^{\ell}\mathbb{R}_{\ell}^{\mathbb{U}} \curvearrowleft {}^{\ell}\mathbb{C}_r^{\mathbb{C}} \end{array} & \xrightarrow{Q} & \begin{array}{c} {}^{=}\mathbb{C}_r^{\mathbb{C}} \\ \downarrow \\ \mathbb{R}^{\mathbb{U}} \curvearrowleft {}^{=}\mathbb{C}_r^{\mathbb{C}} \end{array} \\
 & & 
 \end{array}$$

$$x = {}^{\zeta}Q = {}^t\zeta \zeta = \overbrace{o \cdot \zeta}^t \underbrace{o \cdot \zeta}$$

$$x = {}^t\vartheta \lambda \vartheta \in \Omega_u^1$$

$$u = {}^t\vartheta \vartheta \in S_{\ell}$$

$$u \dot{u} = {}^t\vartheta \underline{\vartheta}^* \bar{\vartheta} = {}^t\vartheta \bar{\vartheta}$$

$$\dot{u} u = {}^*\vartheta \underline{\vartheta}^t \bar{\vartheta} = {}^*\vartheta \bar{\vartheta}$$

$$x \dot{u} = {}^t\vartheta \lambda \underline{\vartheta}^* \bar{\vartheta} = {}^t\vartheta \lambda \bar{\vartheta} = \zeta \bar{\zeta}$$

$$\dot{u} x = {}^*\vartheta \underline{\vartheta}^t \lambda^2 \bar{\vartheta} = {}^*\vartheta \lambda^2 \bar{\vartheta} = \zeta \bar{\zeta}$$

$$\dot{u} x \dot{u} = {}^*\vartheta \underline{\vartheta}^t \lambda^2 \bar{\vartheta} = {}^*\vartheta \lambda^2 \bar{\vartheta} = \zeta \bar{\zeta}$$

$$\dim_{\mathbb{C}} Z_{\ell} = \ell + \frac{\ell(\ell-1)}{2} + \ell(r-\ell) = \ell r - \frac{\ell(\ell-1)}{2}$$

$$\dim_{\mathbb{C}} {}^{\ell}\mathbb{C}_{\ell}^{\mathbb{C}} = \frac{\ell(\ell-1)}{2}$$