

$$\mathcal{O}=\mathfrak{r}\rtimes G\overset{\iota}{\rightarrow}\overset{+}{\mathfrak{g}}\overset{\jmath}{\rightarrow}\mathbb{R}$$

$$\mathfrak{r}\underline{\jmath}\in\mathfrak{g}$$

$$\underline{\mathcal{O}}^{\mathfrak{r}}=\frac{\mathfrak{r}\rtimes\mathfrak{d}}{\mathfrak{d}\in\mathfrak{g}}=\mathfrak{r}\rtimes\mathfrak{g}$$

$$\underbrace{\mathfrak{r}\rtimes\mathfrak{d}}_{\mathfrak{r}}\Omega\,\underbrace{\mathfrak{r}\rtimes\tilde{\mathfrak{d}}}_{\mathfrak{r}}=\mathfrak{r}\underbrace{\mathfrak{d}\rtimes\tilde{\mathfrak{d}}}_{\mathfrak{r}}$$

$$\mathcal{O}\overset{\iota}{\rightarrow}\overset{+}{\mathfrak{g}}$$

$$\underline{\iota\rtimes J^+}^{\mathfrak{r}}=\mathfrak{r}\rtimes\mathfrak{r}\underline{\jmath}$$

$$\underbrace{\mathfrak{r}\rtimes\mathfrak{d}}_{\mathfrak{r}}\mathfrak{r}\mathfrak{r}\underline{\iota\rtimes J^+}^{\mathfrak{r}}=\underbrace{\mathfrak{r}\rtimes\mathfrak{d}}_{\mathfrak{r}}\underline{\iota\rtimes J}=\underbrace{\mathfrak{r}\rtimes\mathfrak{d}}_{\mathfrak{r}}\mathfrak{r}\mathfrak{r}\underline{\jmath}=\mathfrak{r}\underbrace{\mathfrak{d}\rtimes\mathfrak{r}\mathfrak{r}\underline{\jmath}}_{\mathfrak{r}}=\underbrace{\mathfrak{r}\rtimes\mathfrak{d}}_{\mathfrak{r}}\mathfrak{r}\mathfrak{r}\underbrace{\mathfrak{r}\rtimes\mathfrak{r}\mathfrak{r}\underline{\jmath}}_{\mathfrak{r}}$$

$$\underline{\iota\rtimes J}\rtimes \underline{\iota\rtimes \mathfrak{d}}=\iota\rtimes \underline{J\rtimes \mathfrak{d}}$$

$$\mathfrak{r}\underbrace{\underline{\iota\rtimes J}\rtimes \underline{\iota\rtimes \mathfrak{d}}}=\underline{\iota\rtimes J^+}^{\mathfrak{r}}\mathfrak{r}\mathfrak{r}\underline{\iota\rtimes \mathfrak{d}}^{\mathfrak{r}}=\underbrace{\mathfrak{r}\rtimes\mathfrak{r}\mathfrak{r}\underline{\jmath}}_{\mathfrak{r}}\mathfrak{r}\mathfrak{r}\underbrace{\mathfrak{r}\rtimes\mathfrak{r}\mathfrak{r}\underline{\jmath}}_{\mathfrak{r}}=\mathfrak{r}\underbrace{\mathfrak{r}\mathfrak{r}\underline{\jmath}\rtimes\mathfrak{r}\mathfrak{r}\underline{\jmath}}_{\mathfrak{r}}=\mathfrak{r}\underbrace{\mathfrak{r}\mathfrak{r}\underline{\jmath}\rtimes\mathfrak{r}\mathfrak{r}\underline{\jmath}}_{\mathfrak{r}}$$

$$\mathfrak{d}=\partial_t\,g_t$$

$$g\rtimes\mathfrak{d}=\partial_t\,\underbrace{gg_tg^{-1}}$$

$$\overline{P\rtimes g}\rtimes\mathfrak{d}\,_{P\rtimes g}\Omega\,\overline{P\rtimes g}\rtimes\tilde{\mathfrak{d}}=P\rtimes g\mathfrak{d}\rtimes\tilde{\mathfrak{d}}$$

$$\overline{P\rtimes g}\rtimes\mathfrak{d}=\partial_t\,\overline{P\rtimes g\rtimes g_t}=\partial_t\,\overline{P\rtimes gg_t}$$

$$\overline{P\rtimes g}\rtimes\tilde{\mathfrak{d}}=\partial_t\,\overline{P\rtimes g\rtimes\tilde{g}_t}=\partial_t\,\overline{P\rtimes g\tilde{g}_t}$$

$$P\rtimes\overline{g\mathfrak{d}\rtimes\tilde{\mathfrak{d}}}=P\overline{g\rtimes\mathfrak{d}\rtimes\tilde{\mathfrak{d}}}=P\overline{g\rtimes\mathfrak{d}\rtimes g\rtimes\tilde{\mathfrak{d}}}$$