$$\alpha_w = \begin{array}{c|c} \sqrt{u} & v\sqrt[\circ]{u} \\ \hline 0 & \sqrt[\circ]{u} \end{array}$$

$$\alpha_w\left(e\right) = \underbrace{\sqrt{u} + v\sqrt[\circ]{u}}_{}\sqrt{u} = u + v = w$$

$$\alpha_w^{-1} = \begin{array}{c|c} \sqrt[\circ]{u} & -\sqrt[\circ]{u}v \\ \hline 0 & \sqrt{u} \end{array}$$

$$\alpha_w^{-1}\left(w\right) = \underbrace{\sqrt[\circ]{u}\overbrace{u+v}^{} - \sqrt[\circ]{u}v}_{}\sqrt[\circ]{u} = \sqrt[\circ]{u}\,u\sqrt[\circ]{u} = e$$

$$\beta_w = \begin{array}{c|c} 1 & -1 \\ \hline 1 & 1 \end{array}\ \alpha_w \begin{array}{c|c} 1 & 1 \\ \hline -1 & 1 \end{array} = \begin{array}{c|c} \overset{*}{w} + e & w - e \\ \hline \overset{*}{w} - e & w + e \end{array}\ \sqrt[\circ]{u}$$

$$\beta_w\left(0\right) = \underbrace{w - e}_{}\ \overbrace{w + e}^{\circ} = z$$

$$\beta_w^{-1} = \begin{array}{c|c} 1 & -1 \\ \hline 1 & 1 \end{array}\ \alpha_w^{-1} \begin{array}{c|c} 1 & 1 \\ \hline -1 & 1 \end{array} = \sqrt[\circ]{u} \begin{array}{c|c} e + w & e - w \\ \hline e - \overset{*}{w} & e + \overset{*}{w} \end{array}$$

$$\beta_{w'}^{-1}\,\beta_w = \sqrt[\circ]{\acute{u}} \begin{array}{c|c} \acute{w} + \overset{*}{w} & w - \acute{w} \\ \hline \overset{*}{w} - \overset{*}{w}' & w + \overset{*}{w}' \end{array}\ \sqrt[\circ]{u} = \begin{array}{c|c} a & b \\ \hline \bar{b} & \bar{a} \end{array}$$

$$\text{LHS} = \sqrt[\circ]{\acute{u}} \begin{array}{c|c} e + \acute{w} & e - \acute{w} \\ \hline e - \overset{*}{w}' & e + \overset{*}{w}' \end{array}\ \begin{array}{c|c} \overset{*}{w} + e & w - e \\ \hline \overset{*}{w} - e & w + e \end{array}\ \sqrt[\circ]{u}$$

$$= \sqrt[\circ]{\acute{u}} \begin{array}{c|c} \underbrace{e + \acute{w}}_{}\underbrace{\overset{*}{w} + e}_{} + \underbrace{e - \acute{w}}_{}\underbrace{\overset{*}{w} - e}_{} & \underbrace{e + \acute{w}}_{}\underbrace{w - e}_{} + \underbrace{e - \acute{w}}_{}\underbrace{w + e}_{} \\ \hline \underbrace{e - \overset{*}{w}'}_{}\underbrace{\overset{*}{w} + e}_{} + \underbrace{e + \overset{*}{w}'}_{}\underbrace{\overset{*}{w} - e}_{} & \underbrace{e - \overset{*}{w}'}_{}\underbrace{w - e}_{} + \underbrace{e + \overset{*}{w}'}_{}\underbrace{w + e}_{} \end{array}\ \sqrt[\circ]{u} = \text{RHS}$$

$$[\beta_{w'}^{-1}\,\beta_w] = \left[\begin{array}{c|c} 1-\sqrt{u}\,\overbrace{\acute{w}+\overset{*}{w}}^{\circ}\sqrt{u} & \sqrt{u}\,\overbrace{\acute{w}+\overset{*}{w}}^{\circ}\sqrt{\acute{u}} \\ \hline \sqrt{\acute{u}}\,\overbrace{\acute{w}+\overset{*}{w}}^{\circ}\sqrt{u} & 1-\sqrt{\acute{u}}\,\overbrace{\acute{w}+\overset{*}{w}}^{\circ}\sqrt{\acute{u}} \end{array}\right]$$

$$a = \sqrt[\circ]{\acute{u}}\,\underbrace{\acute{w}+\overset{*}{w}}\,\sqrt[\circ]{u}$$

$$b = \sqrt[\circ]{\acute{u}}\,\underbrace{w-\acute{w}}\,\sqrt[\circ]{u}$$

$$a^{-1} = \sqrt{u}\,\overbrace{\acute{w}+\overset{*}{w}}^{\circ}\,\sqrt{\acute{u}}$$

$$\overset{*}{a}^{-1} = \sqrt{\acute{u}}\,\overbrace{\acute{w}+\overset{*}{w}}^{\circ}\,\sqrt{u}$$

$$-\,a^{-1}b = \sqrt{u}\,\overbrace{\acute{w}+\overset{*}{w}}^{\circ}\,\sqrt{\acute{u}}\sqrt[\circ]{\acute{u}}\,\underbrace{\acute{w}-w}\,\sqrt[\circ]{u} = \sqrt{u}\,\overbrace{\acute{w}+\overset{*}{w}}^{\circ}\,\underbrace{\acute{w}-w}\,\sqrt[\circ]{u}$$

$$= \sqrt{u}\,\overbrace{\acute{w}+\overset{*}{w}}^{\circ}\,\underbrace{\acute{w}+\overset{*}{w}-u}\,\sqrt[\circ]{u} = \sqrt{u}\,\underbrace{1-\overbrace{\acute{w}+\overset{*}{w}}^{\circ}u}\,\sqrt[\circ]{u} = 1-\sqrt{u}\,\overbrace{\acute{w}+\overset{*}{w}}^{\circ}\,\sqrt{u}$$

$$\bar{b}\;a^{-1} = \sqrt[\circ]{\acute{u}}\,\underbrace{\overset{*}{w}-\overset{*}{w}'}\,\sqrt[\circ]{u}\sqrt{u}\,\overbrace{\acute{w}+\overset{*}{w}}^{\circ}\,\sqrt{\acute{u}} = \sqrt[\circ]{\acute{u}}\,\underbrace{\overset{*}{w}-\overset{*}{w}'}\,\overbrace{\acute{w}+\overset{*}{w}}^{\circ}\,\sqrt{\acute{u}}$$

$$= \sqrt[\circ]{\acute{u}}\,\underbrace{\acute{w}+\overset{*}{w}-\acute{u}}\,\overbrace{\acute{w}+\overset{*}{w}}^{\circ}\,\sqrt{\acute{u}} = \sqrt[\circ]{\acute{u}}\,\underbrace{1-\acute{u}\,\overbrace{\acute{w}+\overset{*}{w}}^{\circ}}\,\sqrt{\acute{u}} = 1-\sqrt{\acute{u}}\,\overbrace{\acute{w}+\overset{*}{w}}^{\circ}\,\sqrt{\acute{u}}$$

$$\begin{array}{ccc} \mathcal{H}_e & \xleftarrow{[\beta_{w'}^{-1}\beta_w]} & \mathcal{H}_e \\ \Big\downarrow{\scriptstyle \hat{\beta}_{w'}} & & \Big\downarrow{\scriptstyle \hat{\beta}_{w}} \\ \mathcal{H}_{\acute{w}} & \xleftarrow[{}^{\acute{w}}\mathcal{B}_w]{} & \mathcal{H}_w \end{array}$$

$${}^{\acute{w}}\mathcal{B}_w = \hat{\beta}_{w'}\,[\beta_{w'}^{-1}\,\beta_w]\,\hat{\beta}_w^{-1}$$