

$$\begin{aligned}
\check{C}C &= \overline{I - z\check{z}^*}^{-1} \\
\zeta_z &= \overline{I - z\check{z}^*}^{-1/2} \zeta - z\bar{\zeta} \\
\gamma_z^{-1}\zeta &= C \zeta - z\bar{\zeta} = C \overline{\check{C}C}^{-1/2} \overline{\check{C}C}^{1/2} \zeta - z\bar{\zeta} = C \overline{\check{C}C}^{-1/2} \overline{I - z\check{z}^*}^{-1/2} \zeta - z\bar{\zeta} = C \overline{\check{C}C}^{-1/2} \zeta_z
\end{aligned}$$

$$\overline{\gamma_z^{-1}\zeta} | \overline{\gamma_z^{-1}\omega} = \zeta_z | \omega_z$$

$$\overline{C \check{C}C}^{-1/2} \text{ unitary}$$

$$\begin{aligned}
\zeta_w &= u^{-1/2} \overline{w + e\zeta + w - e\bar{\zeta}} \\
&= \begin{array}{c|c} 1 - u^{1/2} \overline{w + \check{w}} u^{1/2} & u^{1/2} \overline{w + \check{w}} u^{1/2} \\ \hline u^{1/2} \overline{w + \check{w}} u^{1/2} & 1 - u^{1/2} \overline{w + \check{w}} u^{1/2} \end{array} \\
&= \begin{bmatrix} \check{w} u^{-1/2} & \check{\zeta} u^{-1/2} \end{bmatrix} \begin{array}{c|c} 1 - u^{1/2} \overline{w + \check{w}} u^{1/2} & u^{1/2} \overline{w + \check{w}} u^{1/2} \\ \hline u^{1/2} \overline{w + \check{w}} u^{1/2} & 1 - u^{1/2} \overline{w + \check{w}} u^{1/2} \end{array} \begin{bmatrix} u^{-1/2} \bar{\alpha} \\ u^{-1/2} \zeta_z \end{bmatrix} \\
&= \begin{bmatrix} \check{w} u^{-1/2} & \check{\zeta} u^{-1/2} \end{bmatrix} \left[ \begin{array}{l} \left( 1 - u^{1/2} \overline{w + \check{w}} u^{1/2} \right) u^{-1/2} \bar{\alpha} + u^{1/2} \overline{w + \check{w}} u^{1/2} u^{-1/2} \zeta_z \\ u^{1/2} \overline{w + \check{w}} u^{1/2} u^{-1/2} \bar{\alpha} + \left( 1 - u^{1/2} \overline{w + \check{w}} u^{1/2} \right) u^{-1/2} \zeta_z \end{array} \right] \\
&= \begin{bmatrix} \check{w} u^{-1/2} & \check{\zeta} u^{-1/2} \end{bmatrix} \left[ \begin{array}{l} \left( u^{-1/2} - u^{1/2} \overline{w + \check{w}} \right) \bar{\alpha} + u^{1/2} \overline{w + \check{w}} \zeta_z \\ u^{1/2} \overline{w + \check{w}} \bar{\alpha} + \left( u^{-1/2} - u^{1/2} \overline{w + \check{w}} \right) \zeta_z \end{array} \right] \\
&= \check{w} u^{-1/2} \left( u^{-1/2} - u^{1/2} \overline{w + \check{w}} \right) \bar{\alpha} + \check{w} u^{-1/2} u^{1/2} \overline{w + \check{w}} \zeta_z + \zeta_8 \\
&\quad + u^{-1/2} u^{1/2} \overline{w + \check{w}} \bar{\alpha} + \check{\zeta} u^{-1/2} \left( u^{-1/2} - u^{1/2} \overline{w + \check{w}} \right) \zeta_z
\end{aligned}$$

$$= \overset{*}{\omega} \overbrace{u^{-1} - \overset{-1}{\dot{w} + \dot{w}^*}} \bar{\alpha} + \overset{*}{\omega} \overbrace{\dot{w} + \dot{w}^*}^{-1} \zeta_z + \overset{+}{\zeta} \overbrace{\dot{w} + \dot{w}^*}^{-1} \bar{\alpha} + \overset{+}{\zeta} \overbrace{u^{-1} - \overset{-1}{\dot{w} + \dot{w}^*}} \zeta_z$$