

$$\begin{array}{ccc}
\mathcal{H}_0 & \xleftarrow{[\gamma_w^{-1}\gamma_z]} & \mathcal{H}_0 \\
\hat{\gamma}_w \downarrow & & \downarrow \hat{\gamma}_z \\
\mathcal{H}_w & \xleftarrow{{}^w\mathcal{B}_z} & \mathcal{H}_z
\end{array}$$

$$\frac{1}{\overset{*}{z}} \left| \begin{array}{c} z \\ 1 \end{array} \right| = \frac{\overbrace{1 - z\overset{*}{z}}^{\circ}}{-\overbrace{1 - \overset{*}{z}z\overset{*}{z}}^{\circ}} \left| \begin{array}{c} -\overbrace{1 - z\overset{*}{z}z}^{\circ} \\ \overbrace{1 - \overset{*}{z}z}^{\circ} \end{array} \right|$$

$$\gamma_z = \frac{1}{\overset{*}{z}} \left| \begin{array}{c} z \\ 1 \end{array} \right| \frac{\overbrace{\sqrt{1 - z\overset{*}{z}}}^{\circ}}{0} \left| \begin{array}{c} 0 \\ \overbrace{\sqrt{1 - \overset{*}{z}z}}^{\circ} \end{array} \right| = \frac{\overbrace{\sqrt{1 - z\overset{*}{z}}}^{\circ}}{0} \left| \begin{array}{c} 0 \\ \overbrace{\sqrt{1 - \overset{*}{z}z}}^{\circ} \end{array} \right| \frac{1}{\overset{*}{z}} \left| \begin{array}{c} z \\ 1 \end{array} \right|$$

$$\text{LHS} = \mathbf{t}_z {}^zB_z^{1/2} \mathbf{t}_z^* = \frac{1}{0} \left| \begin{array}{c} z \\ 1 \end{array} \right| \frac{\overbrace{\sqrt{1 - z\overset{*}{z}}}^{\circ}}{0} \left| \begin{array}{c} 0 \\ \overbrace{\sqrt{1 - \overset{*}{z}z}}^{\circ} \end{array} \right| \frac{1}{\overset{*}{z}} \left| \begin{array}{c} 0 \\ 1 \end{array} \right| = \frac{\overbrace{\sqrt{1 - z\overset{*}{z}}}^{\circ}}{\overbrace{\sqrt{1 - \overset{*}{z}z\overset{*}{z}}}^{\circ}} \left| \begin{array}{c} \overbrace{\sqrt{1 - z\overset{*}{z}z}}^{\circ} \\ \overbrace{\sqrt{1 - \overset{*}{z}z}}^{\circ} \end{array} \right| = \text{RHS}$$

$$\gamma_w^{-1} \gamma_z = \frac{\overbrace{\sqrt{1 - w\overset{*}{w}}\overbrace{1 - w\overset{*}{z}}^{\circ}}^{\circ}}{\overbrace{\sqrt{1 - \overset{*}{w}w}\overbrace{\overset{*}{z} - \overset{*}{w}}^{\circ}}^{\circ}} \frac{\overbrace{\sqrt{1 - z\overset{*}{z}}}^{\circ}}{\overbrace{\sqrt{1 - \overset{*}{z}z}}^{\circ}} \left| \begin{array}{c} \overbrace{\sqrt{1 - w\overset{*}{w}}\overbrace{z - w}}^{\circ} \\ \overbrace{\sqrt{1 - \overset{*}{w}w}\overbrace{1 - \overset{*}{w}z}}^{\circ} \end{array} \right| \frac{\overbrace{\sqrt{1 - \overset{*}{z}z}}^{\circ}}{\overbrace{\sqrt{1 - \overset{*}{z}z}}^{\circ}}$$

$$\begin{aligned}
\text{LHS} &= \gamma_{-w} \gamma_z = \frac{\overbrace{\sqrt{1 - w\overset{*}{w}}}^{\circ}}{0} \left| \begin{array}{c} 0 \\ \overbrace{\sqrt{1 - \overset{*}{w}w}}^{\circ} \end{array} \right| \frac{1}{-\overset{*}{w}} \left| \begin{array}{c} -w \\ 1 \end{array} \right| \frac{1}{\overset{*}{z}} \left| \begin{array}{c} z \\ 1 \end{array} \right| \frac{\overbrace{\sqrt{1 - z\overset{*}{z}}}^{\circ}}{0} \left| \begin{array}{c} 0 \\ \overbrace{\sqrt{1 - \overset{*}{z}z}}^{\circ} \end{array} \right| \\
&= \frac{\overbrace{\sqrt{1 - w\overset{*}{w}}}^{\circ}}{0} \left| \begin{array}{c} 0 \\ \overbrace{\sqrt{1 - \overset{*}{w}w}}^{\circ} \end{array} \right| \frac{1 - w\overset{*}{z}}{\overset{*}{z} - \overset{*}{w}} \left| \begin{array}{c} z - w \\ 1 - \overset{*}{w}z \end{array} \right| \frac{\overbrace{\sqrt{1 - z\overset{*}{z}}}^{\circ}}{0} \left| \begin{array}{c} 0 \\ \overbrace{\sqrt{1 - \overset{*}{z}z}}^{\circ} \end{array} \right| = \text{RHS}
\end{aligned}$$

$$\begin{aligned}
[\gamma_w^{-1} \gamma_z] &= \frac{\sqrt{1-z\overset{\circ}{z}} \sqrt{1-w\overset{*}{z}} \sqrt{w-z} \sqrt{1-\overset{\circ}{z}z}}{\sqrt{1-w\overset{*}{w}} \sqrt{1-\overset{*}{z}w} \sqrt{1-\overset{*}{z}z}} \left| \frac{\sqrt{1-z\overset{\circ}{z}} \sqrt{1-w\overset{*}{z}} \sqrt{1-w\overset{*}{w}}}{\sqrt{1-\overset{*}{w}w} \sqrt{\overset{*}{z}-\overset{*}{w}} \sqrt{1-w\overset{*}{z}} \sqrt{1-w\overset{*}{w}}} \right| \\
&= \frac{{}^z B_z^{-1/2} \left(w-z+Q_{w-z} z^w \right)}{\sqrt{1-\overset{*}{w}w} \sqrt{1-\overset{*}{z}w} \sqrt{1-\overset{*}{z}z}} \left| \frac{\sqrt{1-z\overset{\circ}{z}} \sqrt{1-w\overset{*}{z}} \sqrt{1-w\overset{*}{w}}}{\overset{*}{w} B_{\overset{*}{w}}^{-1/2} \left(\overset{*}{z}-\overset{*}{w}+P_{\overset{*}{z}-\overset{*}{w}} w^z \right)} \right|
\end{aligned}$$

$$a = \sqrt{1-w\overset{*}{w}} \sqrt{1-w\overset{*}{z}} \sqrt{1-\overset{\circ}{z}z}$$

$$\Rightarrow a^{-1} = \sqrt{1-z\overset{\circ}{z}} \sqrt{1-w\overset{*}{z}} \sqrt{1-w\overset{*}{w}}: \quad \bar{a}^{-1} = \sqrt{1-\overset{*}{w}w} \sqrt{1-\overset{*}{z}w} \sqrt{1-\overset{\circ}{z}z}$$

$$b = \sqrt{1-w\overset{*}{w}} \sqrt{z-w} \sqrt{1-\overset{\circ}{z}z} \Rightarrow \bar{b} = \sqrt{1-\overset{*}{w}w} \sqrt{\overset{*}{z}-\overset{*}{w}} \sqrt{1-\overset{\circ}{z}z}$$

$$\sqrt{1-z\overset{\circ}{z}} \sqrt{1-w\overset{*}{z}} \sqrt{w-z} = \left(1-w\overset{*}{z} + \sqrt{w-z\overset{*}{z}} \right) \sqrt{1-w\overset{*}{z}} \sqrt{w-z}$$

$$= w-z + \sqrt{w-z\overset{*}{z}} \sqrt{1-w\overset{*}{z}} \sqrt{w-z} = w-z + \sqrt{w-z\overset{*}{z}} \sqrt{1-z\overset{*}{w}} \sqrt{w-z} = w-z + Q_{w-z} z^w$$

$$\begin{aligned}
-a^{-1}b &= \sqrt{1-z\overset{\circ}{z}} \sqrt{1-w\overset{*}{z}} \sqrt{1-w\overset{*}{w}} \sqrt{1-\overset{\circ}{z}z} \sqrt{w-z} \sqrt{1-\overset{\circ}{z}z} = \sqrt{1-z\overset{\circ}{z}} \sqrt{1-w\overset{*}{z}} \sqrt{w-z} \sqrt{1-\overset{\circ}{z}z} \\
&= \sqrt{1-\overset{\circ}{z}z} \left(\sqrt{1-z\overset{\circ}{z}} \sqrt{1-w\overset{*}{z}} \sqrt{w-z} \right) \sqrt{1-\overset{\circ}{z}z}
\end{aligned}$$

$$= \sqrt{1-\overset{\circ}{z}z} \left(w-z+Q_{w-z} z^w \right) \sqrt{1-\overset{\circ}{z}z} = {}^z B_z^{-1/2} \left(w-z+Q_{w-z} z^w \right)$$

$$\sqrt{\overset{*}{z}-\overset{*}{w}} \sqrt{1-w\overset{*}{z}} \sqrt{1-w\overset{*}{w}} = \sqrt{\overset{*}{z}-\overset{*}{w}} \sqrt{1-w\overset{*}{z}} \left(1-w\overset{*}{z} + \sqrt{w-z\overset{*}{z}} \right)$$

$$= \overset{*}{z}-\overset{*}{w} + \sqrt{\overset{*}{z}-\overset{*}{w}} \sqrt{1-w\overset{*}{z}} \sqrt{w-z\overset{*}{z}} = \overset{*}{z}-\overset{*}{w} + \sqrt{\overset{*}{z}-\overset{*}{w}} w^z \sqrt{\overset{*}{z}-\overset{*}{w}} = \overset{*}{z}-\overset{*}{w} + P_{\overset{*}{z}-\overset{*}{w}} w^z$$

$$\bar{b} a^{-1} = \sqrt{1-\overset{*}{w}w} \sqrt{\overset{*}{z}-\overset{*}{w}} \sqrt{1-\overset{\circ}{z}z} \sqrt{1-z\overset{\circ}{z}} \sqrt{1-w\overset{*}{z}} \sqrt{1-w\overset{*}{w}}$$

$$= \sqrt{1-\overset{*}{w}w} \sqrt{\overset{*}{z}-\overset{*}{w}} \sqrt{1-w\overset{*}{z}} \sqrt{1-w\overset{*}{w}} = \sqrt{1-\overset{*}{w}w} \left(\sqrt{\overset{*}{z}-\overset{*}{w}} \sqrt{1-w\overset{*}{z}} \sqrt{1-w\overset{*}{w}} \right) \sqrt{1-\overset{*}{w}w}$$

$$= \sqrt{1-\overset{*}{w}w} \left(\overset{*}{z}-\overset{*}{w} + P_{\overset{*}{z}-\overset{*}{w}} w^z \right) \sqrt{1-\overset{*}{w}w} = \overset{*}{w} B_{\overset{*}{w}}^{-1/2} \left(\overset{*}{z}-\overset{*}{w} + P_{\overset{*}{z}-\overset{*}{w}} w^z \right)$$

$${}^x\mathcal{B}_y^w = {}^wB_w^{-1/2} {}^x\mathcal{E}_{z-w+Q_{z-w}w^z} {}^wB_w^{1/2} {}^x\mathcal{J}_{zB_w^{-1}zB_z^{1/2}y} {}^{w-z+Q_{w-z}z^w}\mathcal{E}_{zB_z^{-1/2}y}$$

$$\begin{aligned}
& \zeta^+ \sqrt{1-\overset{\circ}{w}w} \left(\overset{*}{z} - \overset{*}{w} + \underbrace{\overset{*}{z} - \overset{*}{w}}_{w^z} \overset{*}{z} - \overset{*}{w} \right) \sqrt{1-\overset{\circ}{w}\overset{*}{w}} \zeta = \text{tr} \zeta \zeta^+ \sqrt{1-\overset{\circ}{w}w} \left(\overset{*}{z} - \overset{*}{w} + \underbrace{\overset{*}{z} - \overset{*}{w}}_{w^z} \overset{*}{z} - \overset{*}{w} \right) \sqrt{1-\overset{\circ}{w}\overset{*}{w}} \\
& = \zeta \zeta^+ \mathfrak{K} \sqrt{1-\overset{\circ}{w}\overset{*}{w}} \underbrace{z-w+\overline{z-w} \overline{w^z} z-w}_{\sqrt{1-\overset{\circ}{w}w}} \sqrt{1-\overset{\circ}{w}\overset{*}{w}} \\
& = x \mathfrak{K} {}^wB_w^{-1/2} \overline{z-w+Q_{z-w}w^z} = {}^wB_w^{-1/2} x \mathfrak{K} \overline{z-w+Q_{z-w}w^z} \\
& \overset{*}{\omega} \sqrt{1-\overset{\circ}{z}\overset{*}{z}} \left(w-z+Q_{w-z}z^w \right) \sqrt{1-\overset{\circ}{z}\overset{*}{z}} \bar{\omega} = \text{tr} \sqrt{1-\overset{\circ}{z}\overset{*}{z}} \left(w-z+Q_{w-z}z^w \right) \sqrt{1-\overset{\circ}{z}\overset{*}{z}} \bar{\omega} \overset{*}{\omega} \\
& = \text{tr} \sqrt{1-\overset{\circ}{z}\overset{*}{z}} \left(w-z+Q_{w-z}z^w \right) \sqrt{1-\overset{\circ}{z}\overset{*}{z}} \overline{\omega^+} = \sqrt{1-\overset{\circ}{z}\overset{*}{z}} \left(w-z+Q_{w-z}z^w \right) \sqrt{1-\overset{\circ}{z}\overset{*}{z}} \mathfrak{K} y \\
& = {}^zB_z^{-1/2} \underbrace{w-z+Q_{w-z}z^w}_{\mathfrak{K} y} = \underbrace{w-z+Q_{w-z}z^w}_{\mathfrak{K} y} {}^zB_z^{-1/2} y \\
& \overset{*}{\omega} \sqrt{1-\overset{\circ}{z}\overset{*}{z}} \overline{1-\overset{\circ}{w}\overset{*}{z}} \sqrt{1-\overset{\circ}{w}\overset{*}{w}} \zeta = \text{tr} \zeta \underbrace{\sqrt{1-\overset{\circ}{w}\overset{*}{w}} \overline{1-\overset{\circ}{z}\overset{*}{w}} \sqrt{1-\overset{\circ}{z}\overset{*}{z}} \omega}_{*} = \langle \zeta | \sqrt{1-\overset{\circ}{w}\overset{*}{w}} \overline{1-\overset{\circ}{z}\overset{*}{w}} \sqrt{1-\overset{\circ}{z}\overset{*}{z}} \omega \rangle \\
& \sqrt{1-\overset{\circ}{w}\overset{*}{w}} \overline{1-\overset{\circ}{z}\overset{*}{w}} \sqrt{1-\overset{\circ}{z}\overset{*}{z}} \omega \underbrace{\sqrt{1-\overset{\circ}{w}\overset{*}{w}} \overline{1-\overset{\circ}{z}\overset{*}{w}} \sqrt{1-\overset{\circ}{z}\overset{*}{z}} \omega}_{+} \\
& = \sqrt{1-\overset{\circ}{w}\overset{*}{w}} \overline{1-\overset{\circ}{z}\overset{*}{w}} \sqrt{1-\overset{\circ}{z}\overset{*}{z}} \omega \overline{\omega^+} \sqrt{1-\overset{\circ}{z}\overset{*}{z}} \overline{1-\overset{\circ}{w}\overset{*}{z}} \sqrt{1-\overset{\circ}{w}\overset{*}{w}} = {}^wB_w^{1/2} {}^zB_w^{-1} {}^zB_z^{1/2} y \\
& \int_{dH}^{\ell_{\mathbb{R}}^U} \mathfrak{e}^{2\langle \zeta[H] | \sqrt{1-\overset{\circ}{w}\overset{*}{w}} \overline{1-\overset{\circ}{z}\overset{*}{w}} \sqrt{1-\overset{\circ}{z}\overset{*}{z}} \omega \rangle} = \zeta \zeta^+ \mathcal{J} \underbrace{\sqrt{1-\overset{\circ}{w}\overset{*}{w}} \overline{1-\overset{\circ}{z}\overset{*}{w}} \sqrt{1-\overset{\circ}{z}\overset{*}{z}} \omega \sqrt{1-\overset{\circ}{w}\overset{*}{w}} \overline{1-\overset{\circ}{z}\overset{*}{w}} \sqrt{1-\overset{\circ}{z}\overset{*}{z}} \omega}_{+} \\
& = {}^x\mathcal{J}_{wB_w^{1/2}zB_w^{-1}zB_z^{1/2}y} = {}^wB_w^{1/2} {}^x\mathcal{J}_{zB_w^{-1}zB_z^{1/2}y}
\end{aligned}$$