

$$\begin{aligned}\hat{g}(x) &= \Phi\left(g\Phi^{-1}x\right): \quad \hat{g}(\Phi x) = \Phi(gx) \\ \hat{\gamma}(x) &= \Phi'\left(\Phi^{-1}x\right)\gamma\left(\Phi^{-1}x\right): \quad \hat{\gamma}(\Phi x) = \Phi'(x)\gamma x\end{aligned}$$

$$\hat{\gamma}_w(x) = 2w\hat{u}x + \alpha\underline{w\star u + u\star w}x \Rightarrow \hat{\rho}_w = 2\partial_{w\hat{u}x} + \alpha\underline{w\star u + u\star w}\partial_x$$

$$\begin{aligned}\gamma_w(u) &= 2w\hat{u}u = 2w_1 + w_{1/2} = \underbrace{w_1 - \hat{w}_1 + w_{1/2}}_{\in T_u(S_1)} + \underbrace{w_1 + \hat{w}_1}_{\in X_u^1} = w_1 - \hat{w}_1 + w_{1/2} + \underline{w\star u + u\star w}u \\ \partial_\varepsilon(t_\varepsilon u_\varepsilon) &= \dot{t}u + t\dot{u} \in T_{tu}(Z_\ell) \\ \gamma_w(tu) &= t\underbrace{w_1 - \hat{w}_1 + w_{1/2}}_{\dot{u} = w_1 - \hat{w}_1 + w_{1/2}} + t\underline{w\star u + u\star w}u \\ \dot{t} &= t\underline{w\star u + u\star w} \\ \Phi(tu) &= t^{1+\alpha}u \\ \Phi'(tu)\underline{\dot{t}u + t\dot{u}} &= \partial_\varepsilon\Phi(t_\varepsilon u_\varepsilon) = \partial_\varepsilon\underbrace{t_\varepsilon^{1+\alpha}u_\varepsilon}_{(1+\alpha)t^\alpha\dot{t}u + t^{1+\alpha}\dot{u}} = (1+\alpha)t^\alpha\dot{t}u + t^{1+\alpha}\dot{u} \in T_{t^{1+\alpha}u}(Z_\ell) \\ \hat{\gamma}_w\left(t^{1+\alpha}u\right) &= \hat{\gamma}_w(\Phi(tu)) = \Phi'(tu)\gamma_w(tu) = (1+\alpha)t^\alpha\underbrace{w\star u + u\star w}_u + t^{1+\alpha}\underbrace{w_1 - \hat{w}_1 + w_{1/2}}_{(1+\alpha)t^{1+\alpha}\underline{w_1 + \hat{w}_1} + t^{1+\alpha}\underline{w_1 - \hat{w}_1 + w_{1/2}}} \\ &= (1+\alpha)t^{1+\alpha}\underline{w_1 + \hat{w}_1} + t^{1+\alpha}\underbrace{w_1 - \hat{w}_1 + w_{1/2}}_{(2+\alpha)w_1 + \alpha\hat{w}_1 + w_{1/2}} = t^{1+\alpha}\left((2+\alpha)w_1 + \alpha\hat{w}_1 + w_{1/2}\right) \\ \hat{\gamma}_w(tu) &= t\left((2+\alpha)w_1 + \alpha\hat{w}_1 + w_{1/2}\right) = t\left(2w_1 + w_{1/2} + \alpha\underbrace{w_1 + \hat{w}_1}_{2w\hat{u}u + \alpha\underline{w\star u + u\star w}u}\right) = t\left(2w\hat{u}u + \alpha\underline{w\star u + u\star w}u\right)\end{aligned}$$