$$a > 0 \begin{cases} x_0 > 0 \\ x_{n+} = \frac{1}{2}\left(x_n + \frac{a}{x_n}\right) > 0 \end{cases} \quad \text{mon fallend/konv/Lim } = \sqrt{a}$$

$$n \geqslant 1 \Longrightarrow x_n^2 \geqslant a$$

$$x^2 + 2a + \frac{a^2}{x^2} - 4a = x^2 - 2a + \frac{a^2}{x^2} = \left(x - \frac{a}{x}\right)^2 \geqslant 0 \Longrightarrow \left(x + \frac{a}{x}\right)^2 = x^2 + 2a + \frac{a^2}{x^2} \geqslant 4a$$

$$x_{n+}^2 = \frac{1}{4}\left(x_n + \frac{a}{x_n}\right)^2 \geqslant a$$

$$x_{n+} \leqslant x_n \text{ antiton}$$

$$\frac{1}{x_n^2} \leqslant \frac{1}{a} \Longrightarrow \frac{x_{n+}}{x_n} = \frac{1}{2x_n}\left(x_n + \frac{a}{x_n}\right) = \frac{1}{2} + \frac{a}{2x_n^2} \leqslant \frac{1}{2} + \frac{a}{2a} = 1$$

$$x_n \rightsquigarrow s \Longrightarrow s^2 = a$$

$$2\,x_{n+}\,x_n = x_n^2 + a \Longrightarrow 2\,s^2 = s^2 + a \Longrightarrow s^2 = a$$

$$a = 2 \begin{cases} x_0 > 0 \\ x_{n+} = \frac{x_n}{2} + \frac{1}{x_n} \end{cases} \quad \text{antiton } x_n \rightsquigarrow \sqrt{2}$$

$\mathbb{Q}$ nicht voll

$$\begin{cases} x_0 = 1/4 \\ x_{n+} = x_n^2 + 1/4 \end{cases} \quad \text{konv/Lim} \quad \begin{cases} x_0 = 0 \\ x_{n+} = 1 + x_n^2 \end{cases} \quad \text{div}$$

$$\begin{cases} x_0 = 1 \\ x_{n+} = \dfrac{2 + x_n}{1 + x_n} \end{cases} \quad \begin{cases} x_0 = 1 \\ x_{n+} = \dfrac{1}{1 + x_n} \end{cases} \quad \text{konv/Lim}$$

$$\begin{cases} x_0 = 0 \\ x_{n+} = \dfrac{5/8}{1 + x_n^2} \end{cases} \quad \text{konv/Lim}$$

$$\begin{cases} 0 < x_0 < a \\ x_{n+} = 2x_n - \dfrac{x_n^2}{a} \end{cases} \quad \text{konv/Lim}$$

$$\begin{cases} a_0 = 1 \\ a_{n+} = 1 + \dfrac{1}{a_n} \end{cases} \Longrightarrow \begin{cases} a_n \leadsto a > 0 \\ \text{bestimme } a \\ \lceil a_n - a \rceil \leqslant \dfrac{1}{a^{n+1}} \end{cases}$$

$$\begin{cases} x_0 = 1 \\ x_{n+} = \sqrt{1 + x_n} \end{cases} \Longrightarrow x_n \leadsto \frac{1 + \sqrt{5}}{2} = \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \cdots}}}} \text{ Kettenwurzel}$$

$$\text{voll } 1|\infty \xrightarrow[\text{contr}]{{}^x\!\curvearrowright = \sqrt{1+x}} 1|\infty \Longrightarrow {}^x\!\curvearrowright - {}^y\!\curvearrowright = \sqrt{1+x} - \sqrt{1+y} = \frac{(1+x) - (1+y)}{\sqrt{1+x} + \sqrt{1+y}} = \frac{x - y}{\sqrt{1+x} + \sqrt{1+y}} \leqslant \frac{\lceil x - y \rceil}{2}$$

$$x_{n+}^2 = 1 + x_n \Longrightarrow s^2 = 1 + s \Longrightarrow s^2 - s - 1 = 0 \Longrightarrow s = \frac{1}{2} + \sqrt{\frac{1}{4} + 1} = \frac{1 + \sqrt{5}}{2}$$

$$\text{konv/Lim} \begin{cases} x_0 = 1 \\ x_{n+} = \sqrt{2x_n} \end{cases}$$

$$a{:}b \in \mathbb{R} \begin{cases} x_0 = a \\ x_1 = b \\ x_{n+} = \dfrac{x_n + x_{n-}}{2} \end{cases}$$