

$$\mathcal{E}_a^m \star \overline{\underline{w} \star \underline{u} \mathcal{E}_b^{m+1}} = c_{m+1} \varrho_{2m+1} {}^a \mathcal{E}_b^m \underline{w} \star \underline{b}$$

$$\text{LHS} = \int \frac{t \varrho}{dt} \int_{du}^{S_1} {}^a \mathcal{E}_{tu}^m \underline{w} \star \underline{u} {}^t \mathcal{E}_b^{m+1} = \int \frac{t \varrho}{dt} t^{2m+1} \int_{du}^{S_1} {}^a \mathcal{E}_u^m \underline{w} \star \underline{u} {}^t \mathcal{E}_b^{m+1} = \varrho_{2m+1} \int_{du}^{S_1} {}^a \mathcal{E}_u^m \underline{w} \star \underline{u} {}^u \mathcal{E}_b^{m+1} = \text{RHS}$$

$$2 \mathcal{E}_a^m \star \overline{\partial_{w \ddot{u} x} {}^u \mathcal{E}_b^{m+1}} = c_{m+1} \varrho_{2m+1} (2m+p) {}^a \mathcal{E}_b^m \underline{w} \star \underline{b}$$

$$\text{LHS} = \int \frac{t \varrho}{dt} t^{2m+1} \int_{du}^{S_1} {}^a \mathcal{E}_u^m \overline{\partial_{w \ddot{u} u} {}^u \mathcal{E}_b^{m+1}} = c_{m+1} (m+p/2) {}^a \mathcal{E}_b^m \underline{w} \star \underline{b} \int \frac{t \varrho}{dt} t^{2m+1} = \text{RHS}$$

$$\mathcal{E}_a^m \star \overline{\underline{w} \star \underline{x} \frac{\varrho}{\varrho} \mathcal{E}_b^{m+1}} = -2(m+1) \varrho_{2m+1} c_{m+1} {}^a \mathcal{E}_b^m \underline{w} \star \underline{b}$$

$$\int \frac{\mathbb{R}^>}{dt} t \varrho t^{2m+2} = t \varrho t^{2m+2} \Big|_0^\infty - 2(m+1) \int \frac{\mathbb{R}^>}{dt} t \varrho t^{2m+1} = -2(m+1) \int \frac{\mathbb{R}^>}{dt} t \varrho t^{2m+1} = -2(m+1) \varrho_{2m+1}$$

$$\text{LHS} = \int \frac{\mathbb{R}^>}{dt} t \varrho t^{2m+2} \int_{du}^{S_1} {}^a \mathcal{E}_u^m \underline{w} \star \underline{u} {}^u \mathcal{E}_b^{m+1} = c_{m+1} {}^a \mathcal{E}_b^m \underline{w} \star \underline{b} \int \frac{\mathbb{R}^>}{dt} t \varrho t^{2m+2} = \text{RHS}$$

$$\overline{\alpha_w \mathcal{E}_b^n} = \frac{c_n \varrho_{2n-1}}{(a/2)_{n-}} \left( n - 1 + \frac{\alpha + p}{2} \right) {}^x \mathcal{E}_b^{n-} \underline{w \blacktriangleright b} - \frac{c_{n+\varrho_{2n+1}}}{(a/2)_{n+}} \left( n + \frac{\alpha + p}{2} \right) {}^x \mathcal{E}_b^n \underline{x \blacktriangleright w}$$

$$\begin{aligned} \text{LHS} &= \mathcal{J}_x \blacktriangleright \overline{\alpha_w \mathcal{E}_b^n} = 2 \mathcal{J}_x \blacktriangleright \overline{\partial_{w \ddot{u} x} \mathcal{E}_b^n} + \mathcal{J}_x \blacktriangleright \overline{w \blacktriangleright u} \left( 1 + \frac{\alpha - p}{2} + \alpha \partial_x \right) \mathcal{E}_b^n + \frac{1 + \alpha}{2} \mathcal{J}_x \blacktriangleright \overline{w \blacktriangleright x} \frac{\varrho}{\varrho} \mathcal{E}_b^n \\ &\quad + \overline{w \blacktriangleright u} \mathcal{J}_x \blacktriangleright \overline{1 + \frac{\alpha - p}{2} + \alpha \partial_x} \mathcal{E}_b^n + \frac{1 + \alpha}{2} \overline{w \blacktriangleright x} \frac{\varrho}{\varrho} \mathcal{J}_x \blacktriangleright \mathcal{E}_b^n = 2 \mathcal{J}_x \blacktriangleright \overline{\partial_{w \ddot{u} x} \mathcal{E}_b^n} \\ &+ \left( 1 + \frac{\alpha - p}{2} + \alpha n \right) \left( \mathcal{J}_x \blacktriangleright \overline{w \blacktriangleright u} \mathcal{E}_b^n + \overline{w \blacktriangleright u} \mathcal{J}_x \blacktriangleright \mathcal{E}_b^n \right) + \frac{1 + \alpha}{2} \left( \mathcal{J}_x \blacktriangleright \overline{w \blacktriangleright x} \frac{\varrho}{\varrho} \mathcal{E}_b^n + \overline{w \blacktriangleright x} \frac{\varrho}{\varrho} \mathcal{J}_x \blacktriangleright \mathcal{E}_b^n \right) \\ &= 2 \frac{\mathcal{E}_x^{n-}}{(a/2)_{n-}} \blacktriangleright \overline{\partial_{w \ddot{u} x} \mathcal{E}_b^n} + \left( 1 + \frac{\alpha - p}{2} + \alpha n \right) \left( \frac{\mathcal{E}_x^{n-}}{(a/2)_{n-}} \blacktriangleright \overline{w \blacktriangleright u} \mathcal{E}_b^n + \overline{w \blacktriangleright u} \frac{\mathcal{E}_x^{n+}}{(a/2)_{n+}} \blacktriangleright \mathcal{E}_b^n \right) \\ &+ \frac{1 + \alpha}{2} \left( \frac{\mathcal{E}_x^{n-}}{(a/2)_{n-}} \blacktriangleright \overline{w \blacktriangleright x} \frac{\varrho}{\varrho} \mathcal{E}_b^n + \overline{w \blacktriangleright x} \frac{\varrho}{\varrho} \frac{\mathcal{E}_x^{n+}}{(a/2)_{n+}} \blacktriangleright \mathcal{E}_b^n \right) = \frac{c_n \varrho_{2n-1}}{(a/2)_{n-}} (2n - 2 + p) {}^x \mathcal{E}_b^{n-} \underline{w \blacktriangleright b} \\ &\quad + \left( 1 + \frac{\alpha - p}{2} + \alpha n \right) \left( \frac{c_n \varrho_{2n-1} {}^x \mathcal{E}_b^{n-} \underline{w \blacktriangleright b}}{(a/2)_{n-}} + \frac{c_{n+\varrho_{2n+1}} {}^x \mathcal{E}_b^n \underline{x \blacktriangleright w}}{(a/2)_{n+}} \right) \\ &\quad - \frac{1 + \alpha}{2} \left( \frac{c_n \varrho_{2n-1} 2n {}^x \mathcal{E}_b^{n-} \underline{w \blacktriangleright b}}{(a/2)_{n-}} + \frac{c_{n+\varrho_{2n+1}} 2(n+1) {}^x \mathcal{E}_b^n \underline{x \blacktriangleright w}}{(a/2)_{n+}} \right) \\ &= \frac{c_n \varrho_{2n-1} {}^x \mathcal{E}_b^{n-} \underline{w \blacktriangleright b}}{(a/2)_{n-}} \left( 2n - 2 + p + 1 + \frac{\alpha - p}{2} + \alpha n - (1 + \alpha) n \right) \\ &\quad + \frac{c_{n+\varrho_{2n+1}} {}^x \mathcal{E}_b^n \underline{x \blacktriangleright w}}{(a/2)_{n+}} \left( 1 + \frac{\alpha - p}{2} + \alpha n - (1 + \alpha) (n + 1) \right) = \text{RHS} \end{aligned}$$

$$\begin{aligned} \overline{\alpha_v \mathcal{E}_a^m} \blacktriangleright \overline{\alpha_w \mathcal{E}_b^n} &= \int_{dx} {}^x \overline{\alpha_v \mathcal{E}_a^m} {}^x \overline{\alpha_w \mathcal{E}_b^n} = \int_{dx} \overline{\alpha_v \mathcal{E}_a^m \blacktriangleright \mathcal{J}_x} \mathcal{J}_x \blacktriangleright \overline{\alpha_w \mathcal{E}_b^n} \\ &= \int_{dx} \frac{c_m \varrho_{2m-1}}{(a/2)_{m-}} \left( m - 1 + \frac{\alpha + p}{2} \right) {}^a \mathcal{E}_x^{m-} \underline{a \blacktriangleright v} \frac{c_n \varrho_{2n-1}}{(a/2)_{n-}} \left( n - 1 + \frac{\alpha + p}{2} \right) {}^x \mathcal{E}_b^{n-} \underline{w \blacktriangleright b} \end{aligned}$$

$$\begin{aligned}
& - \int_{dx} \frac{c_m \varrho_{2m-1}}{(a/2)_{m-}} \left( m - 1 + \frac{\alpha + p}{2} \right) {}^a \mathcal{E}_x^{m-} \underline{a \star v} \frac{c_{n+\varrho_{2n+1}}}{(a/2)_{n+}} \left( n + \frac{\alpha + p}{2} \right) {}^x \mathcal{E}_b^n \underline{x \star w} \\
& - \int_{dx} \frac{c_{m+\varrho_{2m+1}}}{(a/2)_{m+}} \left( m + \frac{\alpha + p}{2} \right) {}^a \mathcal{E}_x^m \underline{v \star x} \frac{c_n \varrho_{2n-1}}{(a/2)_{n-}} \left( n - 1 + \frac{\alpha + p}{2} \right) {}^x \mathcal{E}_b^{n-} \underline{w \star b} \\
& + \int_{dx} \frac{c_{m+\varrho_{2m+1}}}{(a/2)_{m+}} \left( m + \frac{\alpha + p}{2} \right) {}^a \mathcal{E}_x^m \underline{v \star x} \frac{c_{n+\varrho_{2n+1}}}{(a/2)_{n+}} \left( n + \frac{\alpha + p}{2} \right) {}^x \mathcal{E}_b^n \underline{x \star w} \\
= & \frac{c_m \varrho_{2m-1}}{(a/2)_{m-}} \frac{c_n \varrho_{2n-1}}{(a/2)_{n-}} \left( m - 1 + \frac{\alpha + p}{2} \right) \left( n - 1 + \frac{\alpha + p}{2} \right) \underline{a \star v} \underline{w \star b} \int_{dx} {}^a \mathcal{E}_x^{m-} \star {}^x \mathcal{E}_b^{n-} \\
& - \frac{c_m \varrho_{2m-1}}{(a/2)_{m-}} \frac{c_{n+\varrho_{2n+1}}}{(a/2)_{n+}} \left( m - 1 + \frac{\alpha + p}{2} \right) \left( n + \frac{\alpha + p}{2} \right) \underline{a \star v} \int_{dx} {}^a \mathcal{E}_x^{m-} {}^x \mathcal{E}_b^n \underline{x \star w} \\
& - \frac{c_{m+\varrho_{2m+1}}}{(a/2)_{m+}} \frac{c_n \varrho_{2n-1}}{(a/2)_{n-}} \left( m + \frac{\alpha + p}{2} \right) \left( n - 1 + \frac{\alpha + p}{2} \right) \underline{w \star b} \int_{dx} {}^a \mathcal{E}_x^m \underline{v \star x} {}^x \mathcal{E}_b^{n-} \\
& + \frac{c_{m+\varrho_{2m+1}}}{(a/2)_{m+}} \frac{c_{n+\varrho_{2n+1}}}{(a/2)_{n+}} \left( m + \frac{\alpha + p}{2} \right) \left( n + \frac{\alpha + p}{2} \right) \int_{dx} {}^a \mathcal{E}_x^m \underline{v \star x} {}^x \mathcal{E}_b^n \underline{x \star w} \\
& \int_{dx} {}^a \mathcal{E}_x^{m-} {}^x \mathcal{E}_b^{n-} = \delta_m^n (a/2)_{m-} {}^a \mathcal{E}_b^{m-} \\
& \int_{dx} {}^a \mathcal{E}_x^{m-} {}^x \mathcal{E}_b^n \underline{x \star w} = c_{m+} {}^a \mathcal{E}_b^m \underline{v \star b} \varrho_{m+n}
\end{aligned}$$