$$\bar{ɓ}\,◺\,\overset{m}{\bar{ɓ}}$$

$$\begin{array}{c} \bar{ɓ}\,◺\,\overset{m}{\bar{ɓ}} \\ \Big\downarrow d \\ \bar{ɓ}\,◺\,\overset{m+1}{\bar{ɓ}} \end{array}$$

$$\begin{bmatrix} {}_0ɓ \\ + \\ {}_mɓ \end{bmatrix} d٩ = \sum_{0 \leqslant i \leqslant m} \overset{i}{-1}\ {}_iɓ \ast \begin{bmatrix} {}_0ɓ \\ + \\ \hat{{}_iɓ} \\ + \\ {}_mɓ \end{bmatrix} ٩ + \sum_{0 \leqslant i < j \leqslant m} \overset{j-i}{-1} \begin{bmatrix} {}_iɓ \ast {}_jɓ \\ {}_0ɓ \\ + \\ \hat{{}_iɓ} \\ + \\ \hat{{}_jɓ} \\ + \\ {}_mɓ \end{bmatrix} ٩$$

$${}_Mɓ\, d٩ = \sum_i^M \underset{M \llcorner i}{\overset{i}{\vee}}\ {}_iɓ \ast {}_{M\llcorner i}ɓ\,٩ + \sum_{i<j} \underset{M \llcorner i}{\overset{i}{\vee}} \underset{M \llcorner j}{\overset{j}{\vee}} \begin{bmatrix} {}_iɓ \ast {}_jɓ \\ {}_{M\llcorner i:j}ɓ \end{bmatrix} ٩$$

$$ɓ\ast = \underbrace{ɓ \vDash} d + d \underbrace{ɓ \vDash}$$

$${}_Mɓ\, \overbrace{ɓ \vDash \underbrace{d٩}} + {}_Mɓ\, \overbrace{\underbrace{ɓ \vDash ٩}} = \begin{bmatrix} ɓ \\ {}_Mɓ \end{bmatrix} \underbrace{d٩} + \sum_i^M \underset{M \llcorner i}{\overset{i}{\vee}}\ {}_iɓ \ast \overbrace{{}_{M\llcorner i}ɓ \underbrace{ɓ \vDash ٩}} + \sum_{i<j} \underset{M \llcorner i}{\overset{i}{\vee}} \underset{M \llcorner j}{\overset{j}{\vee}} \begin{bmatrix} {}_iɓ \ast {}_jɓ \\ {}_{M\llcorner i:j}ɓ \end{bmatrix} \underbrace{ɓ \vDash ٩}$$

$$= ɓ \ast \overbrace{{}_Mɓ٩} - \sum_i^M \underset{M \llcorner i}{\overset{i}{\vee}}\ {}_iɓ \ast \begin{bmatrix} ɓ \\ {}_{M\llcorner i}ɓ \end{bmatrix} ٩ - \sum_i^M \underset{M \llcorner i}{\overset{i}{\vee}} \begin{bmatrix} ɓ \ast {}_iɓ \\ {}_{M\llcorner i}ɓ \end{bmatrix} ٩ + \sum_{i<j} \underset{M \llcorner i}{\overset{i}{\vee}} \underset{M \llcorner j}{\overset{j}{\vee}} \begin{bmatrix} {}_iɓ \ast {}_jɓ \\ ɓ \\ {}_{M\llcorner i:j}ɓ \end{bmatrix} ٩$$

$$+ \sum_i^M \underset{M \llcorner i}{\overset{i}{\vee}}\ {}_iɓ \ast \begin{bmatrix} ɓ \\ {}_{M\llcorner i}ɓ \end{bmatrix} ٩ + \sum_{i<j} \underset{M \llcorner i}{\overset{i}{\vee}} \underset{M \llcorner j}{\overset{j}{\vee}} \begin{bmatrix} ɓ \\ {}_iɓ \ast {}_jɓ \\ {}_{M\llcorner i:j}ɓ \end{bmatrix} ٩ = ɓ \ast \overbrace{ɓ_M ٩} + \sum_i^M \underset{M \llcorner i}{\overset{i}{\vee}} \begin{bmatrix} {}_iɓ \ast ɓ \\ {}_{M\llcorner i}ɓ \end{bmatrix} ٩ = {}_Mɓ\, \underbrace{ɓ \ast ٩}$$

$$\underbrace{\mathsf{b}\times}d = d\underbrace{\mathsf{b}\times}$$

$$\underbrace{\mathsf{b}\times}d = \overbrace{\underbrace{\mathsf{b}\vDash}d + d\underbrace{\mathsf{b}\vDash}}d = d\left(-\mathsf{b}\vDash\right)d = d\overbrace{\underbrace{\mathsf{b}\vDash}d + d\underbrace{\mathsf{b}\vDash}} = d\underbrace{\mathsf{b}\times}$$

$$dd = 0$$

$$\mathsf{b}\vDash\overbrace{d\underbrace{d\mathsf{q}}} = \mathsf{b}\times\underbrace{d\mathsf{q}} - d\overbrace{\mathsf{b}\vDash\underbrace{d\mathsf{q}}} = d\underbrace{\mathsf{b}\times\mathsf{q}} - d\overbrace{\mathsf{b}\vDash\underbrace{d\mathsf{q}}} = d\overbrace{\mathsf{b}\times\mathsf{q} - \mathsf{b}\vDash\underbrace{d\mathsf{q}}} = d\overbrace{d\underbrace{\mathsf{b}\vDash\mathsf{q}}} = dd\underbrace{\mathsf{b}\vDash\mathsf{q}} \underset{\text{Ind}}{=} 0$$