$$0 \leqslant x < 1 \Longrightarrow \begin{cases} x^n\,(1-x) < \dfrac{1}{n} \\ x^n(1-x)^k < \dfrac{k!}{n^k} \end{cases}$$

$$1 = 1^{n+1} = \overbrace{\underbrace{1-x}_{} + x}^{n+1} \geqslant \begin{bmatrix} n+1 \\ 1 \end{bmatrix} \overbrace{1-x}\, x^n > (1-x)\, n\, x^n$$

$$\begin{bmatrix} n+k \\ k \end{bmatrix} = \frac{(n+k)!}{k!n!} = \frac{(n+1)\cdot\cdot(n+k)}{k!} > \frac{n^k}{k!}$$

$$1 = 1^{n+k} = \overbrace{\underbrace{1-x}_{} + x}^{n+k} \geqslant \begin{bmatrix} n+k \\ k \end{bmatrix} \overbrace{1-x}^{k}\, x^n > \frac{n^k}{k!}\, x^n\, \overbrace{1-x}^{k}$$

$$n^k\, x^n \rightsquigarrow 0: \quad x^n \rightsquigarrow 0$$

$$n^k\, x^n = \frac{n^{k+1}}{n}\, x^n < \frac{(k+1)!}{(1-x)^{k+1}}\, \frac{1}{n} \rightsquigarrow 0$$

$$x^n = \frac{n}{n}\, x^n < \frac{1}{1-x}\, \frac{1}{n} \rightsquigarrow 0$$

$$\text{geom seq } \overline{x} < 1 \Longrightarrow x^n \rightsquigarrow 0$$

$$\text{geom sum } x \neq 1 \Longrightarrow \sum_{m}^{n} x^m = \frac{1-x^n}{1-x}$$

$$\underline{0 \leqslant n \curvearrowright n+1}: \quad \sum_{m \leqslant n} x^m = \sum_{m}^{n} x^m + x^n \underset{\text{ind}}{=} \frac{1-x^n}{1-x} + x^n\, \frac{1-x}{1-x}$$

$$= \frac{\overbrace{1-x^n} + x^n \overbrace{1-x}}{1-x} = \frac{1-x^n+x^n-x^{n+1}}{1-x} = \frac{1-x^{n+1}}{1-x}$$

$$\text{geom ser } \overline{x} < 1 \Longrightarrow \sum_{m}^{n} x^m \rightsquigarrow \frac{1}{1-x} = {\overbrace{1-x}}^{-1}$$