$$\mathbb{H}\times\mathbb{F}\sim\mathbb{H}\times\mathbb{E}\Leftrightarrow\bigvee_{\mathbb{H}\times\mathbb{J}}\mathbb{H}\times\underbrace{\mathbb{F}+\mathbb{J}}\asymp\mathbb{H}\times\underbrace{\mathbb{E}+\mathbb{J}}$$

$$\mathbb{H}\times\mathbb{J}\sim\mathbb{H}\times\mathbb{F}\sim\mathbb{H}\times\mathbb{E}\underset{\text{trans}}{\Longrightarrow}\mathbb{H}\times\mathbb{J}\sim\mathbb{H}\times\mathbb{E}$$

$$\begin{cases}\mathbb{H}\times\underbrace{\mathbb{J}+\mathbb{J}}\asymp\mathbb{H}\times\underbrace{\mathbb{F}+\mathbb{J}}\\ \mathbb{H}\times\underbrace{\mathbb{F}+\mathbb{I}}\asymp\mathbb{H}\times\underbrace{\mathbb{E}+\mathbb{I}}\end{cases}$$

$$\Longrightarrow\mathbb{H}\times\overbrace{\mathbb{J}+\underbrace{\mathbb{J}+\mathbb{F}+\mathbb{I}}}=\mathbb{H}\times\overbrace{\underbrace{\mathbb{J}+\mathbb{J}}+\underbrace{\mathbb{F}+\mathbb{I}}}\asymp\mathbb{H}\times\overbrace{\underbrace{\mathbb{F}+\mathbb{J}}+\underbrace{\mathbb{E}+\mathbb{I}}}=\mathbb{H}\times\overbrace{\mathbb{E}+\underbrace{\mathbb{J}+\mathbb{F}+\mathbb{I}}}$$

$$\mathbb{H}\times\mathbb{F}\sim\mathbb{H}\times\mathbb{E}\Longrightarrow\dim\mathbb{H}\times\mathbb{F}=\dim\mathbb{H}\times\mathbb{E}$$

$$\mathbb{H}\times\underbrace{\mathbb{F}+\mathbb{J}}\sim\mathbb{H}\times\underbrace{\mathbb{E}+\mathbb{J}}\underset{\text{cancel}}{\Longrightarrow}\mathbb{H}\times\mathbb{F}\sim\mathbb{H}\times\mathbb{E}$$

$$\mathbb{H}\times\underbrace{\mathbb{F}+\mathbb{J}}\sim\mathbb{H}\times\underbrace{\mathbb{E}+\mathbb{J}}\Longrightarrow\mathbb{H}\times\overbrace{\underbrace{\mathbb{F}+\mathbb{J}}+\mathbb{J}}\sim\mathbb{H}\times\overbrace{\underbrace{\mathbb{E}+\mathbb{J}}+\mathbb{J}}$$

$$\Longrightarrow\mathbb{H}\times\overbrace{\mathbb{F}+\underbrace{\mathbb{J}+\mathbb{J}}}=\mathbb{H}\times\overbrace{\underbrace{\mathbb{F}+\mathbb{J}}+\mathbb{J}}\sim\mathbb{H}\times\overbrace{\underbrace{\mathbb{E}+\mathbb{J}}+\mathbb{J}}\sim\mathbb{H}\times\overbrace{\mathbb{E}+\underbrace{\mathbb{J}+\mathbb{J}}}$$

$$\begin{cases}\mathbb{H}\times\mathbb{F}\sim\mathbb{H}\times\mathbb{E}\\ \mathbb{H}\times\mathbb{F}\sim\mathbb{H}\times\mathbb{E}\end{cases}\Longrightarrow\mathbb{H}\times\underbrace{\mathbb{F}+\mathbb{F}}\sim\mathbb{H}\times\underbrace{\mathbb{E}+\mathbb{E}}$$

$$\begin{cases}\mathbb{H}\times\underbrace{\mathbb{F}+\mathbb{J}}\sim\mathbb{H}\times\underbrace{\mathbb{E}+\mathbb{J}}\\ \mathbb{H}\times\underbrace{\mathbb{F}+\mathbb{J}}\sim\mathbb{H}\times\underbrace{\mathbb{E}+\mathbb{J}}\end{cases}$$

$$\Longrightarrow\mathbb{H}\times\overbrace{\underbrace{\mathbb{F}+\mathbb{F}}+\underbrace{\mathbb{J}+\mathbb{J}}}=\mathbb{H}\times\overbrace{\underbrace{\mathbb{F}+\mathbb{J}}+\underbrace{\mathbb{F}+\mathbb{J}}}\sim\mathbb{H}\times\overbrace{\underbrace{\mathbb{E}+\mathbb{J}}+\underbrace{\mathbb{E}+\mathbb{J}}}=\mathbb{H}\times\overbrace{\underbrace{\mathbb{E}+\mathbb{E}}+\underbrace{\mathbb{J}+\mathbb{J}}}$$

$$\begin{cases} ㅂ \times ㅈ \sim ㅂ \times ㅌ \\ ㅂ \times ㅈ \sim ㅂ \times ㅌ \end{cases} \Rightarrow ㅂ \times \underbrace{ㅈ \boxtimes ㅈ} \sim ㅂ \times \underbrace{ㅌ \boxtimes ㅌ}$$

$$\begin{cases} ㅂ \times \underbrace{ㅈ + ㅁ} \sim ㅂ \times \underbrace{ㅌ + ㅁ} \\ ㅂ \times \underbrace{ㅈ + ㅁ} \sim ㅂ \times \underbrace{ㅌ + ㅁ} \end{cases}$$

$$\Rightarrow ㅂ \times \overbrace{\underbrace{ㅈ \times ㅈ} + \underbrace{ㅈ \times ㅁ} + \underbrace{ㅁ \times ㅈ} + \underbrace{ㅁ \times ㅁ}} = ㅂ \times \overbrace{\underbrace{ㅈ + ㅁ} \times \underbrace{ㅈ + ㅁ}}$$

$$\sim ㅂ \times \overbrace{\underbrace{ㅌ + ㅁ} \times \underbrace{ㅌ + ㅁ}} = ㅂ \times \overbrace{\underbrace{ㅌ \times ㅌ} + \underbrace{ㅌ \times ㅁ} + \underbrace{ㅁ \times ㅌ} + \underbrace{ㅁ \times ㅁ}}$$