



$$X^u \underline{\pi} = 0 \Rightarrow X = A^e \underline{u \otimes} \Rightarrow X^u \omega = A \Rightarrow X - \underline{X^u \omega}^e \underline{u \otimes} = A^e \underline{u \otimes} - A^e \underline{u \otimes} = 0 \Rightarrow {}^u \underline{\mathbb{H}}_{u \xi} \text{ well-def}$$

$${}^u \underline{\xi} {}^u \xi \underline{\pi} = {}^u \widehat{\underline{\xi} \pi} = {}^u v \mapsto v \xi \mapsto \pi(v \xi) = v \bar{T} = {}^u \underline{\pi}$$

$$\underline{X^u \omega}^e \underline{u \otimes} {}^u \underline{\pi} = \underline{X^u \omega}^e \widehat{\underline{u \otimes} \pi} = \underline{X^u \omega}^e \underline{\mathbb{H}} \mapsto u \otimes \mathbb{H} \mapsto \pi(u \otimes \mathbb{H}) = u \bar{T} = 0$$

$$\begin{array}{ccc}
\mathbb{H}_{u\bar{H}} & \xrightarrow{\quad u\bar{H} - u\bar{H}\mathbb{H}_{u\bar{H}} \quad} & \frac{u\bar{H} \times |\bar{H}|}{u\lambda^{u\bar{H}}} \\
& \searrow \begin{matrix} u\widetilde{d\bar{H}} + u\bar{\lambda} \bowtie u\widetilde{H} \\ \text{---} \end{matrix} & \nearrow \begin{matrix} u\widetilde{H} \\ u\lambda \end{matrix} \\
& |\bar{H}| &
\end{array}$$

$$\begin{aligned}
X^u \underline{\pi}^{\widehat{u\bar{H} - u\bar{H}\mathbb{H}_{u\bar{H}}}} &= X \widehat{id \times d\bar{H}}^{u\lambda^{u\bar{H}}} - X^u \underline{\pi}^{\widehat{u\bar{H}\mathbb{H}_{u\lambda^{u\bar{H}}}}} = \widehat{X : X^{\widehat{d\bar{H}}}}^{u\lambda^{u\bar{H}}} - \widehat{X - X^u \omega^e u \bowtie u\bar{H}} \\
&= X^u \underline{\lambda^{u\bar{H}}} + X^u \widehat{d\bar{H}}^{u\lambda^{u\bar{H}}} - X^u \underline{\lambda^{u\bar{H}}} + \widehat{X^u \omega^e u \bowtie u\bar{H}} \\
&= X^u \widehat{d\bar{H}}^{u\lambda^{u\bar{H}}} + \widehat{X^u \omega^e u \bowtie u\bar{H}} = \widehat{X^u \bar{d\bar{H}}} + \widehat{X^u \omega^e u \bowtie u\bar{H}}
\end{aligned}$$

$$\begin{array}{ccccc}
& & u\bar{H} \widehat{d + \omega \bowtie \bar{H}} & & \\
& \swarrow & & \searrow & \\
\mathbb{H}_{u\bar{H}} & \xrightarrow{\quad u\bar{H} - u\bar{H}\mathbb{H}_{u\bar{H}} \quad} & \frac{u\bar{H} \times |\bar{H}|}{u\lambda^{u\bar{H}}} & \xleftarrow{\tau_{u\lambda^{u\bar{H}}}} & u\bar{H} \times |\bar{H}| \\
\uparrow & & \downarrow u\widetilde{H} & & \uparrow u\lambda \\
& \searrow & & \nearrow u\lambda & \\
\mathbb{H} \times \mathbb{H}_u & \xrightarrow{\quad u\widetilde{d\bar{H}} + u\omega \bowtie u\widetilde{H} \quad} & |\bar{H}| & &
\end{array}$$

$$\begin{array}{ccc}
\frac{\underline{\mathbb{h}} \times \underline{\mathbb{h}}}{u} & \xrightarrow{u_\omega \bowtie u \tilde{\mathbb{H}}} & |\underline{\mathbb{h}}| \\
\downarrow^{u \underline{\mathbf{x}} \underline{\mathbb{h}}} & & \downarrow \underline{\mathbb{h}}^{-\bowtie} \\
\frac{\underline{\mathbb{h}} \times \underline{\mathbb{h}}}{u^{\mathbb{h}}} & \xrightarrow{u^{\mathbb{h}} \omega \bowtie u^{\mathbb{h}} \tilde{\mathbb{H}}} & |\underline{\mathbb{h}}|
\end{array}$$

$$u \underline{\mathbf{x}} \underline{\mathbb{h}} \underbrace{u^{\mathbb{h}} \omega \bowtie u^{\mathbb{h}} \tilde{\mathbb{H}}} = \underline{\mathbb{h}}^{-\bowtie} \underbrace{u_\omega \bowtie u \tilde{\mathbb{H}}}$$

$$u \underline{\mathbf{x}} \underline{\mathbb{h}} \underbrace{u^{\mathbb{h}} \omega \bowtie u^{\mathbb{h}} \tilde{\mathbb{H}}} = \widehat{u \underline{\mathbf{x}} \underline{\mathbb{h}} u^{\mathbb{h}} \omega} \bowtie u^{\mathbb{h}} \tilde{\mathbb{H}} = \widehat{u_\omega e \underline{\mathbf{x}} \underline{\mathbb{h}}} \bowtie u^{\mathbb{h}} \tilde{\mathbb{H}} = \underline{\mathbb{h}}^{-\bowtie} \widehat{u_\omega \bowtie \underline{\mathbb{h}}^{\bowtie u \tilde{\mathbb{H}}}} = \underline{\mathbb{h}}^{-\bowtie} \underbrace{u_\omega \bowtie u \tilde{\mathbb{H}}}$$

$$\ker u_{\underline{\pi}} | \widehat{d \tilde{\mathbb{H}}} + u_\omega \bowtie u \tilde{\mathbb{H}} = 0$$

$$\ker u_{\underline{\pi}} \ni A^e \underline{u \mathbf{x}} \Rightarrow \widehat{A^e \underline{u \mathbf{x}} u \omega} \bowtie u \tilde{\mathbb{H}} = A \bowtie u \tilde{\mathbb{H}}$$

$$\widehat{A^e \underline{u \mathbf{x}}} \widehat{d \tilde{\mathbb{H}}} = A^e \underline{u \mathbf{x}} u \tilde{\mathbb{H}} = A^e \widehat{\underline{u \mathbf{x}} \tilde{\mathbb{H}}} = A^e \underline{\mathbb{h} \mapsto u \mathbf{x} \mathbb{h} \mapsto u \tilde{\mathbb{H}}} = \underline{\mathbb{h}^{-\bowtie} u \tilde{\mathbb{H}}} = -A \bowtie u \tilde{\mathbb{H}}$$

$$\underline{\mathbb{h}}_{u \tilde{\mathbb{H}}} \ni \mathbb{X}^u p \curvearrowright X \widehat{d \tilde{\mathbb{H}}} + u_\omega \bowtie u \tilde{\mathbb{H}} = X \widehat{d \tilde{\mathbb{H}}} + \widehat{X^u \omega} \bowtie u \tilde{\mathbb{H}} \text{ well-def}$$

$$\begin{array}{ccc}
& \xrightarrow{\quad u \widetilde{d}^{\mathbb{H}} + u \omega \bowtie u \widetilde{\mathbb{H}} \quad} & \\
\mathbb{H} \times \mathbb{H}_u & \xrightarrow{\quad u \pi_{-} \quad} & |\mathbb{H}| \\
\downarrow & \nearrow u \underline{\mathbb{X}} \mathbb{H} & \downarrow \mathbb{H}^{-\bowtie} \\
& \mathbb{H}_{u \mathbb{H}} & \\
\downarrow & \nearrow u \underline{\mathbb{H}} \pi_{-} & \downarrow u \ell \\
\mathbb{H} \times \mathbb{H}_{u \mathbb{H}} & \xrightarrow{\quad u \underline{\mathbb{H}} \widetilde{d}^{\mathbb{H}} + u \mathbb{H} \omega \bowtie u \underline{\mathbb{H}} \widetilde{\mathbb{H}} \quad} & |\mathbb{H}| \\
& \xrightarrow{\quad u \mathbb{H} \widetilde{d}^{\mathbb{H}} + u \omega \bowtie u \widetilde{\mathbb{H}} \quad} & \\
& \xrightarrow{\quad u \mathbb{H} \widetilde{d}^{\mathbb{H}} + u \omega \bowtie u \widetilde{\mathbb{H}} \quad} & u \mathbb{H} \times |\mathbb{H}| \\
\mathbb{H}_{u \mathbb{H}} & \xrightarrow{\quad u \pi_{-} \quad} & u \mathbb{H} \times |\mathbb{H}| \\
\uparrow & \nearrow u \ell & \uparrow u \ell \\
\mathbb{H} \times \mathbb{H}_u & \xrightarrow{\quad u \widetilde{d}^{\mathbb{H}} + u \omega \bowtie u \widetilde{\mathbb{H}} \quad} & |\mathbb{H}|
\end{array}$$

$\mathbb{H} \times \mathbb{H}_u \ni X \nmid X^{u \pi_{-}} \xrightarrow{u \mathbb{H} \widetilde{d}^{\mathbb{H}} + u \omega \bowtie u \widetilde{\mathbb{H}}} = u \ell \underbrace{X^{u \widetilde{d}^{\mathbb{H}}} + \overline{X^u \omega \bowtie u \widetilde{\mathbb{H}}}}$