

$$\begin{array}{ccccc}
 & & \varrho_k^K & & \\
 & \mathcal{I}^2(K) & \longleftarrow & \mathcal{I}^2(K) & \\
 & \uparrow \mathfrak{i}_K & & \uparrow \mathfrak{i}_K & \\
 P_K & \curvearrowright & & \curvearrowright & P_K \\
 & {}^L\mathcal{I}^2(K) & \longleftarrow \varrho_k^K & {}^L\mathcal{I}^2(K) & \\
 & \uparrow \mathfrak{j} & & \uparrow \mathfrak{j} & \\
 \mathfrak{j}^* & \curvearrowright & & \curvearrowright & \mathfrak{j}^* \\
 & \mathcal{I}^2(S) & \longleftarrow \varrho_k^S & \mathcal{I}^2(S) & \\
 & \uparrow \mathfrak{i}_S & & \uparrow \mathfrak{i}_S & \\
 P_S & \curvearrowright & & \curvearrowright & P_S \\
 & \mathcal{O}^2(S) & \longleftarrow \varrho_k^S & \mathcal{O}^2(S) &
 \end{array}$$

$${}^k\overline{P_K F} = \int_{d\ell}^L \ell^k F$$

$${}^s\overline{P_S f} = \int_{dt}^S e^{\Delta_t^{-d/r} \, t} f$$

$${}^k\widehat{\mathfrak{j}f} = {}^{ek}f$$

$$\int_{ds}^S {}^s\overline{f} \, {}^s\widehat{\mathfrak{j}^*F} = f \, \overline{\mathfrak{J}_S^*F} = \mathfrak{j}f \, \overline{\mathfrak{J}_K^*F} = \int_{dk}^K {}^k\overline{\mathfrak{j}f} \, {}^kF = \int_{dk}^K {}^{ek}\overline{f} \, {}^kF$$

$$\mathfrak{i}_K \mathfrak{j} \mathfrak{i}_S P_S \mathfrak{j}^* P_K \in W_\ell^*(K)$$

$$\mathfrak{i}_K \mathfrak{j}_S P_S \mathfrak{j}^* P_K = \int\limits_{dE\left(k\right)}^K \ell_k$$

$$dE\left(h\right)=dh\,{}^{eh}\Delta_e^{-d/r}$$

$$\begin{aligned} \overbrace{\mathfrak{i}_K \mathfrak{j}_S P_S \mathfrak{j}^* P_K F}^k &= \overbrace{\mathfrak{j}_S P_S \mathfrak{j}^* P_K F}^k = \overbrace{\mathfrak{i}_S P_S \mathfrak{j}^* P_K F}^{ek} = \overbrace{P_S \mathfrak{j}^* P_K F}^{ek} \\ &= \int\limits_{ds}^S {}^{ek}\Delta_s^{-d/r} \overbrace{\mathfrak{j}^* P_K F}^s = \int\limits_{ds}^S {}^s\overline{\Delta_{ek}}^{-d/r} \overbrace{\mathfrak{j}^* P_K F}^s \\ &= \int\limits_{dk}^K {}^{ek}\overline{\Delta_{ek}}^{-d/r} \overbrace{P_K F}^k = \int\limits_{dk}^K {}^{ek}\Delta_{ek}^{-d/r} \int\limits_{d\ell}^L {}^{\ell k} F = \int\limits_{d\ell}^L \int\limits_{dk}^K {}^{ek}\Delta_{ek}^{-d/r} {}^{\ell k} F \\ &= \int\limits_{d\ell}^L \int\limits_{dh}^K {}^{ek}\Delta_{e\mathring{h}k}^{-d/r} {}^{\mathring{h}k} F = \int\limits_{d\ell}^L \int\limits_{dh}^K {}^e\Delta_{e\mathring{h}}^{-d/r} {}^{\mathring{h}k} F = \int\limits_{dh}^K {}^e\Delta_{e\mathring{h}}^{-d/r} {}^{\mathring{h}k} F \\ &\qquad \int\limits_{dE\left(k\right)}^K \overbrace{\ell_k F}^k = \int\limits_{dE\left(k\right)}^K {}^{k\circ k} F \end{aligned}$$