$$\prod_j \left(\lambda + t_j\right) = \det\left(\lambda + t\right) = \det\left(\lambda + \overset{1/2}{\eta}\,\widetilde{\eta}\,\overset{1/2}{\eta}\right) = \det \overset{1/2}{\eta} \det\left(\lambda \mathring{\eta} + \widetilde{\eta}\right) \det \overset{1/2}{\eta} = \det \eta \det\left(\lambda \mathring{\eta} + \widetilde{\eta}\right)$$

$$= \det\left(\lambda + \eta\widetilde{\eta}\right) = \det\left(\lambda + \underbrace{s + iy}\,\underbrace{s - iy}\right) = \det\left(\lambda + s^2 + y^2 + iy \times s\right)$$

$$\frac{\partial}{\partial s_i} \prod_j \left(\lambda + t_j\right) = \sum_k \frac{\partial\left(\lambda + t_k\right)}{\partial s_i} \prod_{j \neq k} \left(\lambda + t_j\right) = \prod_j \left(\lambda + t_j\right) \sum_k \frac{1}{\lambda + t_k} \frac{\partial t_k}{\partial s_i}$$

$$\dot{x}\ {}^{x}\underline{\det} = \ \mathrm{tr}\ \left(\mathring{x}\dot{x}\right)\ {}^{x}\det \ = \ \mathring{x}\,|\,\dot{x}\ {}^{x}\det$$

$$\frac{\partial}{\partial s_i} \det\left(\lambda + s^2 + y^2 + iy \times s\right) = \frac{\partial}{\partial s_i}\left(\lambda + s^2 + y^2 + iy \times s\right)\ {}^{\lambda + s^2 + y^2 + iy \times s}\underline{\det}$$

$$= \ \overbrace{\lambda + s^2 + y^2 + iy \times s}^{\circ}\,|\,\underbrace{\frac{\partial}{\partial s_i}\left(s^2 + iy \times s\right)} \det\left(\lambda + s^2 + y^2 + iy \times s\right)$$

$$\frac{\partial}{\partial s_i}\left(s^2 + iy \times s\right) = 2\,s_i\,e_i + iy \times e_i$$

$$\Rightarrow \sum_k \frac{1}{\lambda + t_k} \frac{\partial t_k}{\partial s_i} = \ \overbrace{\lambda + \eta\widetilde{\eta}}^{\circ}\,|\,\underbrace{\frac{\partial}{\partial s_i}\left(s^2 + iy \times s\right)} \ = \ \overbrace{\lambda + \eta\widetilde{\eta}}^{\circ}\,|\,\underbrace{2s_i e_i + iy \times e_i}$$

$$\left.\begin{array}{c|c|c|c} \dfrac{\partial t_1}{\partial s_1} & \dfrac{\partial t_\ell}{\partial s_1} & \dfrac{1}{\lambda_1 + t_1} & \dfrac{1}{\lambda_\ell + t_1} \\ \hline \\ \hline \dfrac{\partial t_1}{\partial s_\ell} & \dfrac{\partial t_\ell}{\partial s_\ell} & \dfrac{1}{\lambda_1 + t_\ell} & \dfrac{1}{\lambda_\ell + t_\ell} \end{array}\right. = \begin{array}{c|c} \overbrace{\lambda_1 + \eta\widetilde{\eta}}^{\circ}\,|\,\underbrace{2s_1 e_1 + iy \times e_1} & \overbrace{\lambda_\ell + \eta\widetilde{\eta}}^{\circ}\,|\,\underbrace{2s_1 e_1 + iy \times e_1} \\ \hline \\ \hline \overbrace{\lambda_1 + \eta\widetilde{\eta}}^{\circ}\,|\,\underbrace{2s_\ell e_\ell + iy \times e_\ell} & \overbrace{\lambda_\ell + \eta\widetilde{\eta}}^{\circ}\,|\,\underbrace{2s_\ell e_\ell + iy \times e_\ell} \end{array}$$

$$\det \begin{array}{c|c} \dfrac{1}{\lambda_1 + t_1} & \dfrac{1}{\lambda_\ell + t_1} \\ \hline \\ \hline \dfrac{1}{\lambda_1 + t_\ell} & \dfrac{1}{\lambda_\ell + t_\ell} \end{array} = \frac{\prod_{i<j} \overbrace{\lambda_i - \lambda_j} \prod_{i<j} \overbrace{t_i - t_j}}{\prod_{i<j} \overbrace{\lambda_i + t_j}}$$