

$$Z = X \times Y \times V = U \times V$$

$$Z_{\mathbb{C}} = U_{\mathbb{C}} \times V_{\mathbb{C}}$$

$$r_{\mathbb{C}} = \operatorname{rank} U_{\mathbb{C}} = \begin{cases} r & ABD \\ 2r & C|BC \end{cases}$$

$$Z_{>} = \frac{u:v \in U \times V}{u + u^* - 2vv^* \in X_{>}}$$

$$\begin{array}{ccc} X & \xrightarrow{\hspace{2cm}} & X^{\mathbb{C}} \\ & \searrow & \downarrow \\ & U & \\ & \searrow & \downarrow \\ X_{\mathbb{C}} & \xrightarrow{\hspace{2cm}} & U_{\mathbb{C}} = X^{\mathbb{C}}_{\mathbb{C}} \end{array}$$

$$U = \frac{u \in U_{\mathbb{C}}}{\widetilde{u} = u}$$

$$X = \frac{x \in U}{x^* = x} = \frac{x_{\mathbb{C}} \in X_{\mathbb{C}}}{\widetilde{x}_{\mathbb{C}} = x_{\mathbb{C}}} = \frac{x \in X^{\mathbb{C}}}{\bar{x} = x}$$

$$X_{\mathbb{C}} = \frac{x_{\mathbb{C}} \in U_{\mathbb{C}}}{x_{\mathbb{C}}^* = x_{\mathbb{C}}}$$

$$X^{\mathbb{C}} = \frac{u_{\mathbb{C}} \in U_{\mathbb{C}}}{\widetilde{u}_{\mathbb{C}} = u_{\mathbb{C}}^*}$$

$$u_{\mathbb{C}}\!:\!v_{\mathbb{C}} \in Z_{\mathbb{C}} \Rightarrow u_{\mathbb{C}} + \tilde{u}_{\mathbb{C}}^* - 2v_{\mathbb{C}} \tilde{v}_{\mathbb{C}}^* \in X_{>}^{\mathbb{C}}$$

$$u_{\mathbb{C}} + \tilde{u}_{\mathbb{C}}^* - 2v_{\mathbb{C}} \tilde{v}_{\mathbb{C}}^* \in X^{\mathbb{C}} \sqsubset U_{\mathbb{C}}$$

$$\begin{aligned} \Re_{\mathbb{C}}\left(u_{\mathbb{C}} + \tilde{u}_{\mathbb{C}}^* - 2v_{\mathbb{C}} \tilde{v}_{\mathbb{C}}^*\right) &= u_{\mathbb{C}} + \tilde{u}_{\mathbb{C}}^* - 2v_{\mathbb{C}} \tilde{v}_{\mathbb{C}}^* + u_{\mathbb{C}}^* + \tilde{u}_{\mathbb{C}} - 2\tilde{v}_{\mathbb{C}} v_{\mathbb{C}}^* \\ &= \underbrace{u_{\mathbb{C}} + u_{\mathbb{C}}^* - 2v_{\mathbb{C}} v_{\mathbb{C}}^*}_{\in X_{\mathbb{C}}^{>}} + \underbrace{\tilde{u}_{\mathbb{C}} + \tilde{u}_{\mathbb{C}}^* - 2\tilde{v}_{\mathbb{C}} \tilde{v}_{\mathbb{C}}^*}_{\in X_{\mathbb{C}}^{>}} + \underbrace{2v_{\mathbb{C}} - \tilde{v}_{\mathbb{C}}}_{\in X_{\mathbb{C}}^{\geqslant}} \underbrace{v_{\mathbb{C}}^* - \tilde{v}_{\mathbb{C}}}_{\in X_{\mathbb{C}}^{\geqslant}} \in X_{\mathbb{C}}^{>} \cap X^{\mathbb{C}} = X_{>} \end{aligned}$$