

$$\begin{array}{c} \overline{\mathfrak{b}} \\ \diagdown \quad \diagup \\ \overline{\mathfrak{b}} \end{array} \begin{matrix} m \\ 1 \end{matrix}$$

d

$$\left[\begin{smallmatrix} \mathfrak{b} & 0 \\ + & \\ m\mathfrak{b} & \end{smallmatrix} \right] d\mathfrak{q} = \sum_{0 \leq i \leq m} (-1)^i \mathfrak{b} \bowtie \left[\begin{smallmatrix} \mathfrak{b} & 0 \\ + & \\ \hat{\mathfrak{b}} & \\ + & \\ m\mathfrak{b} & \end{smallmatrix} \right] \mathfrak{q} + \sum_{0 \leq i < j \leq m} (-1)^{j-i} \left[\begin{smallmatrix} \mathfrak{b} & \mathfrak{b} \\ \mathfrak{b} & \\ 0 & \\ + & \\ \hat{\mathfrak{b}} & \\ i & \\ + & \\ \hat{\mathfrak{b}} & \\ j & \\ + & \\ m\mathfrak{b} & \end{smallmatrix} \right] \mathfrak{q}$$

$${}_M \mathfrak{b} d\mathfrak{q} = \sum_i^M {}_{M \sqcup i} \mathfrak{b} \bowtie {}_{M \sqcup i} \mathfrak{b} \mathfrak{q} + \sum_{i < j} {}_{M \sqcup i} \mathfrak{b} {}_{M \sqcup j} \left[\begin{smallmatrix} \mathfrak{b} & \mathfrak{b} \\ \mathfrak{b} & \\ M \sqcup i:j & \end{smallmatrix} \right] \mathfrak{q}$$

$$\mathfrak{b} \bowtie = \underline{\mathfrak{b}} \underline{\vdash} d + d \underline{\mathfrak{b}} \underline{\vdash}$$

$$\begin{aligned} {}_M \mathfrak{b} \widehat{\mathfrak{b} \vdash d \mathfrak{q}} + {}_M \mathfrak{b} \widehat{\mathfrak{b} \vdash \mathfrak{q}} &= \left[\begin{smallmatrix} \mathfrak{b} \\ M\mathfrak{b} \end{smallmatrix} \right] \widehat{d\mathfrak{q}} + \sum_i^M {}_{M \sqcup i} \mathfrak{b} \bowtie \widehat{{}_{M \sqcup i} \mathfrak{b} \widehat{\mathfrak{b} \vdash \mathfrak{q}}} + \sum_{i < j} {}_{M \sqcup i} \mathfrak{b} {}_{M \sqcup j} \left[\begin{smallmatrix} \mathfrak{b} & \mathfrak{b} \\ \mathfrak{b} & \\ M \sqcup i:j & \end{smallmatrix} \right] \widehat{\mathfrak{b} \vdash \mathfrak{q}} \\ &= \mathfrak{b} \bowtie \widehat{{}_M \mathfrak{b} \mathfrak{q}} - \sum_i^M {}_{M \sqcup i} \mathfrak{b} \bowtie \left[\begin{smallmatrix} \mathfrak{b} \\ M \sqcup i \mathfrak{b} \end{smallmatrix} \right] \mathfrak{q} - \sum_i^M {}_{M \sqcup i} \left[\begin{smallmatrix} \mathfrak{b} & \mathfrak{b} \\ M \sqcup i \mathfrak{b} & \end{smallmatrix} \right] \mathfrak{q} + \sum_{i < j} {}_{M \sqcup i} \mathfrak{b} {}_{M \sqcup j} \left[\begin{smallmatrix} \mathfrak{b} & \mathfrak{b} \\ \mathfrak{b} & \\ M \sqcup i:j & \end{smallmatrix} \right] \mathfrak{q} \\ &+ \sum_i^M {}_{M \sqcup i} \mathfrak{b} \bowtie \left[\begin{smallmatrix} \mathfrak{b} \\ M \sqcup i \mathfrak{b} \end{smallmatrix} \right] \mathfrak{q} + \sum_{i < j} {}_{M \sqcup i} \mathfrak{b} {}_{M \sqcup j} \left[\begin{smallmatrix} \mathfrak{b} & \mathfrak{b} \\ \mathfrak{b} & \\ M \sqcup i:j & \end{smallmatrix} \right] \mathfrak{q} = \mathfrak{b} \bowtie \widehat{{}_M \mathfrak{b} \mathfrak{q}} + \sum_i^M {}_{M \sqcup i} \left[\begin{smallmatrix} \mathfrak{b} & \mathfrak{b} \\ \mathfrak{b} & \end{smallmatrix} \right] \mathfrak{q} = {}_M \mathfrak{b} \widehat{\mathfrak{b} \bowtie \mathfrak{q}} \end{aligned}$$

$$\underline{b} \times d = d \underline{b} \times$$

$$\underline{b} \times d = \overbrace{\underline{b} \times d + d \underline{b} \times} = d(-\underline{b} \times) d = d \overbrace{\underline{b} \times d + d \underline{b} \times} = d \underline{b} \times$$

$$dd = 0$$

$$\underline{b} \times \overbrace{dd \underline{q}} = \underline{b} \times \underline{d} \underline{q} - d \overbrace{\underline{b} \times \underline{d} \underline{q}} = d \underline{b} \times \underline{q} - d \overbrace{\underline{b} \times \underline{d} \underline{q}} = d \overbrace{\underline{b} \times \underline{q} - \underline{b} \times \underline{d} \underline{q}} = d \overbrace{d \underline{b} \times \underline{q}} = dd \underline{b} \times \underline{q} \stackrel{\text{Ind}}{=} 0$$