$$u = \frac{1 - z\bar{z}}{1 + z\bar{z}}$$

$$v = \frac{z + \bar{z}}{1 + z\bar{z}}$$

$$w = i\frac{\bar{z} - z}{1 + z\bar{z}}$$

$$av + bw + \varrho u = a\frac{z + \bar{z}}{1 + z\bar{z}} + bi\frac{\bar{z} - z}{1 + z\bar{z}} + \varrho\frac{1 - z\bar{z}}{1 + z\bar{z}}$$

$$= \frac{(a - ib)\, z + (a + ib)\, \bar{z} + \varrho\left(1 - z\bar{z}\right)}{1 + z\bar{z}} = \frac{\sigma z + \tau\bar{z} + \varrho\left(1 - z\bar{z}\right)}{1 + z\bar{z}}$$

$$= \frac{1}{1 + z\bar{z}} \begin{bmatrix} 1 & z \end{bmatrix} \left[\begin{array}{c|c} \varrho & \sigma \\ \hline \tau & -\varrho \end{array}\right] \begin{bmatrix} 1 \\ \bar{z} \end{bmatrix}$$

$$\sigma\tau = (a + ib)\,(a - ib) = a^2 + b^2 = -\varrho^2$$

$$z|1\left[\begin{array}{c|c} a & b \\ \hline c & d \end{array}\right] = za + c|zb + d = \frac{za + c}{zb + d}$$