$$X^n_{\mathbb{C}} \text{ normal variety}$$

$$\dim_{\mathbb{C}} X = n$$

$$\text{line bundles } \begin{cases} \bigwedge^n \underline{X}^+ = K_X & \text{canonical} \\ \bigwedge^n \underline{X} = -K_X & \text{anti-canonical} \end{cases}$$

$$X^n_{\mathbb{C}} \begin{cases} \text{canonical=general type} \\ \text{anti-canonical=Fano} \end{cases} \Leftrightarrow \text{ ample } \begin{cases} K_X & \text{can divisor} \\ -K_X & \text{anti-can divisor} \end{cases}$$

$$X^2_{\mathbb{C}} \text{ anti-canonical=del Pezzo}$$

$$\text{Pic } X^2_{\mathbb{C}}$$

$$C \text{ regular/du Val singular}$$

$$\text{Serre dual } C + \widetilde{C} = -K$$

$$\text{gen}_C = 1 - \frac{C|\widetilde{C}}{2} = 1 + \frac{C|(K+C)}{2} = \text{gen}_{\widetilde{C}}$$

$$C \text{ rat } \Leftrightarrow \text{gen}_C = 0 \Leftrightarrow C|\widetilde{C} = 2 \Leftrightarrow \widetilde{C} \text{ rat}$$

$$-K \text{ ample } \Rightarrow -K| - K = K|K \geqslant 0$$

$$\deg_C = -K|C$$

$$\deg_C + \deg_{\widetilde{C}} = -K|\underbrace{C + \widetilde{C}} = -K| - K = K|K$$

$$0 \leqslant p \leqslant q = K|K - p$$

$$\text{rat}^p_q \begin{cases} C \text{ rat} & \deg_C = p \\ \widetilde{C} \text{ rat} & \deg_{\widetilde{C}} = q \end{cases}$$