$$\frac{\sigma z+\tau z^{*}+\varrho\left(1-zz^{*}\right)}{1+z\bar{z}}=\frac{1}{1+zz^{*}}\begin{bmatrix}1 & z\end{bmatrix}\left[\begin{array}{c|c}\varrho & \sigma\\ \hline \tau & -\varrho\end{array}\right]\begin{bmatrix}1\\ z^{*}\end{bmatrix}$$

$$\sigma\tau=\left(a+ib\right)\left(a-ib\right)=a^{2}+b^{2}=-\varrho^{2}$$

$$\begin{bmatrix}1 & z\end{bmatrix}\left[\begin{array}{c|c}d & b\\ \hline c & a\end{array}\right]=\begin{bmatrix}d+zc & b+za\end{bmatrix}=\underbrace{d+zc}\begin{bmatrix}1 & \overbrace{d+zc}^{-1}\underbrace{b+za}\end{bmatrix}$$

$$\begin{bmatrix}1 & z\end{bmatrix}\left[\begin{array}{c|c}\varrho & \sigma\\ \hline \tau & -\varrho\end{array}\right]\begin{bmatrix}1\\ z^{*}\end{bmatrix}=\begin{bmatrix}\varrho+z\tau & \sigma-z\varrho\end{bmatrix}\begin{bmatrix}1\\ z^{*}\end{bmatrix}=\varrho+z\tau+\overbrace{\sigma-z\varrho}z^{*}=\varrho+z\tau+\sigma\,z^{*}-z\varrho\,z^{*}$$

$$\left[\begin{array}{c|c}d & b\\ \hline c & a\end{array}\right]\left[\begin{array}{c|c}d^{*} & c^{*}\\ \hline b^{*} & a^{*}\end{array}\right]=\left[\begin{array}{c|c}dd^{*}+bb^{*} & dc^{*}+ba^{*}\\ \hline cd^{*}+ab^{*} & cc^{*}+aa^{*}\end{array}\right]=\left[\begin{array}{c|c}1 & 0\\ \hline 0 & 1\end{array}\right]$$

$$\left[\begin{array}{c|c}d^{*} & c^{*}\\ \hline b^{*} & a^{*}\end{array}\right]\left[\begin{array}{c|c}d & b\\ \hline c & a\end{array}\right]=\left[\begin{array}{c|c}d^{*}d+c^{*}c & d^{*}b+c^{*}a\\ \hline b^{*}d+a^{*}c & b^{*}b+a^{*}a\end{array}\right]=\left[\begin{array}{c|c}1 & 0\\ \hline 0 & 1\end{array}\right]$$

$$\overbrace{d+zc}^{-1}\underbrace{b+za}=\underbrace{d^{*}z-c^{*}}\overbrace{a^{*}-b^{*}z}^{-1}$$

$$\underbrace{b+za}\underbrace{a^{*}-b^{*}z}=\underbrace{d+zc}\underbrace{d^{*}z-c^{*}}$$

$$1+\overbrace{d+zc}^{-1}\underbrace{b+za}\left(\overbrace{d+zc}^{-1}\underbrace{b+za}\right)^{*}=\overbrace{d+zc}^{-1}\underbrace{1+zz^{*}}\overbrace{d^{*}+c^{*}z^{*}}^{-1}$$

$$\text{LHS}=1+\overbrace{d+zc}^{-1}\underbrace{b+za}\underbrace{b^{*}+a^{*}z^{*}}\overbrace{d^{*}+c^{*}z^{*}}^{-1}=\overbrace{d+zc}^{-1}\overbrace{\underbrace{d+zc}\underbrace{d^{*}+c^{*}z^{*}}+\underbrace{b+za}\underbrace{b^{*}+a^{*}z^{*}}}\overbrace{d^{*}+c^{*}z^{*}}^{-1}=\text{RHS}$$

$$\begin{bmatrix}1 & \overbrace{d+zc}^{-1}\underbrace{b+za}\end{bmatrix}\left[\begin{array}{c|c}\varrho & \sigma\\ \hline \tau & -\varrho\end{array}\right]\begin{bmatrix}1\\ \left(\overbrace{d+zc}^{-1}\underbrace{b+za}\right)^{*}\end{bmatrix}$$

$$\overbrace{d+zc}^{-1}\begin{bmatrix}1 & z\end{bmatrix}\left[\begin{array}{c|c}d & b\\ \hline c & a\end{array}\right]\left[\begin{array}{c|c}\varrho & \sigma\\ \hline \tau & -\varrho\end{array}\right]\left[\begin{array}{c|c}d^{*} & c^{*}\\ \hline b^{*} & a^{*}\end{array}\right]\begin{bmatrix}1\\ z^{*}\end{bmatrix}\overbrace{d^{*}+c^{*}z^{*}}^{-1}$$