$$\begin{cases} \text{left action} \\ \text{right action} \end{cases}$$

$$\overset{s:t}{\overleftarrow{\varphi \boldsymbol{\mathsf{x}} \psi}} = {}^{ts}\varphi\,{}^{t}\psi$$

$$\overset{s:t}{\overrightarrow{\varphi \boldsymbol{\mathsf{x}} \psi}} = {}^{st}\varphi\,{}^{t}\psi$$

$$\underbrace{\overleftarrow{g} \boldsymbol{\mathsf{x}} \overleftarrow{g}}_{\lrcorner} = 1 \boldsymbol{\mathsf{x}} \overleftarrow{g} \implies \underbrace{\overleftarrow{u}}_{\lrcorner} = 1 \boldsymbol{\mathsf{x}} \overleftarrow{u}$$

$$\underbrace{\overrightarrow{g} \boldsymbol{\mathsf{x}} \overrightarrow{g}}_{\llcorner} = 1 \boldsymbol{\mathsf{x}} \overrightarrow{g} \implies \underbrace{\overrightarrow{u}}_{\llcorner} = 1 \boldsymbol{\mathsf{x}} \overrightarrow{u}$$

$$\overset{s:t}{\overleftarrow{\underbrace{\overleftarrow{g} \boldsymbol{\mathsf{x}} \overleftarrow{g}}\, \underbrace{\varphi \boldsymbol{\mathsf{x}} \psi}}} = \overset{s:t}{\overleftarrow{\overleftarrow{g} \varphi \boldsymbol{\mathsf{x}} \overleftarrow{g} \psi}} = \overset{ts}{\overleftarrow{\overleftarrow{g} \varphi}}\,\overset{t}{\overleftarrow{\overleftarrow{g} \psi}} = {}^{g^{-}ts}\varphi\,{}^{g^{-}t}\psi = \overset{s:g^{-}t}{\overleftarrow{\varphi \boldsymbol{\mathsf{x}} \psi}} = \overset{s:t}{\overleftarrow{\underbrace{1 \boldsymbol{\mathsf{x}} \overleftarrow{g}}\, \overleftarrow{\varphi \boldsymbol{\mathsf{x}} \psi}}}$$

$$\overset{s:t}{\overrightarrow{\underbrace{\overrightarrow{g} \boldsymbol{\mathsf{x}} \overrightarrow{g}}\, \underbrace{\varphi \boldsymbol{\mathsf{x}} \psi}}} = \overset{s:t}{\overrightarrow{\overrightarrow{g} \varphi \boldsymbol{\mathsf{x}} \overrightarrow{g} \psi}} = \overset{st}{\overrightarrow{\overrightarrow{g} \varphi}}\,\overset{t}{\overrightarrow{\overrightarrow{g} \psi}} = {}^{stg}\varphi\,{}^{tg}\psi = \overset{s:tg}{\overrightarrow{\varphi \boldsymbol{\mathsf{x}} \psi}} = \overset{s:t}{\overrightarrow{\underbrace{1 \boldsymbol{\mathsf{x}} \overrightarrow{g}}\, \overrightarrow{\varphi \boldsymbol{\mathsf{x}} \psi}}}$$

$$\overset{s:t}{\lrcorner}\!\urcorner = {}^{ts}\urcorner \implies \underbrace{\urcorner \boldsymbol{\mathsf{x}} 1}_{\lrcorner} = \lrcorner\urcorner \text{ bi-mult}$$

$$\overset{s:t}{\llcorner}\!\urcorner = {}^{st}\urcorner \implies \underbrace{\urcorner \boldsymbol{\mathsf{x}} 1}_{\llcorner} = \llcorner\urcorner \text{ bi-mult}$$

$$\overset{s:t}{\overleftarrow{\underbrace{\urcorner \boldsymbol{\mathsf{x}} 1}\, \underbrace{\varphi \boldsymbol{\mathsf{x}} \psi}}} = \overset{s:t}{\overleftarrow{\underbrace{\urcorner \varphi}\, \boldsymbol{\mathsf{x}} \psi}} = \overset{ts}{\overleftarrow{\urcorner \varphi}}\,{}^{t}\psi = {}^{ts}\urcorner\,{}^{ts}\varphi\,{}^{t}\psi = \overset{s:t}{\lrcorner}\urcorner\,\overset{s:t}{\overleftarrow{\varphi \boldsymbol{\mathsf{x}} \psi}} = \overset{s:t}{\overleftarrow{\lrcorner\urcorner\, \varphi \boldsymbol{\mathsf{x}} \psi}}$$

$$\overset{s:t}{\overrightarrow{\underbrace{\urcorner \boldsymbol{\mathsf{x}} 1}\, \underbrace{\varphi \boldsymbol{\mathsf{x}} \psi}}} = \overset{s:t}{\overrightarrow{\underbrace{\urcorner \varphi}\, \boldsymbol{\mathsf{x}} \psi}} = \overset{st}{\overrightarrow{\urcorner \varphi}}\,{}^{t}\psi = {}^{st}\urcorner\,{}^{st}\varphi\,{}^{t}\psi = \overset{s:t}{\llcorner}\urcorner\,\overset{s:t}{\overrightarrow{\varphi \boldsymbol{\mathsf{x}} \psi}} = \overset{s:t}{\overrightarrow{\llcorner\urcorner\, \varphi \boldsymbol{\mathsf{x}} \psi}}$$

$$\underbrace{\lrcorner\urcorner \boldsymbol{\mathsf{x}} 1}_{{}_{\imath}\boldsymbol{\mathsf{x}}\lrcorner} = \underbrace{\lrcorner\urcorner}_{\llcorner\boldsymbol{\mathsf{x}} 1}$$

$$\underbrace{\llcorner\urcorner \boldsymbol{\mathsf{x}} 1}_{{}_{\imath}\boldsymbol{\mathsf{x}}\llcorner} = \underbrace{\llcorner\urcorner}_{\lrcorner\,\boldsymbol{\mathsf{x}} 1}$$

$$\mathbb{1}\boldsymbol{\mathsf{x}} \overset{\mathbb{H}}{\underset{0}{\triangleright}} \mathbb{C} \ni \lrcorner\urcorner = b_{i} \boldsymbol{\mathsf{x}} \urcorner^{i} \implies \underbrace{b \boldsymbol{\mathsf{x}} \urcorner \boldsymbol{\mathsf{x}} 1}_{{}_{\imath}\boldsymbol{\mathsf{x}}\lrcorner} = b \boldsymbol{\mathsf{x}} \lrcorner\urcorner = \underbrace{b \boldsymbol{\mathsf{x}} \urcorner}_{{}_{\imath}\boldsymbol{\mathsf{x}}\lrcorner} \implies \text{LHS} = \underbrace{\lrcorner\urcorner}_{{}_{\imath}\boldsymbol{\mathsf{x}}\lrcorner} = \text{RHS}$$

$$\mathbb{1}\boldsymbol{\mathsf{x}} \overset{\mathbb{H}}{\underset{0}{\triangleright}} \mathbb{C} \ni \llcorner\urcorner = b_{i} \boldsymbol{\mathsf{x}} \urcorner^{i} \implies \underbrace{b \boldsymbol{\mathsf{x}} \urcorner \boldsymbol{\mathsf{x}} 1}_{{}_{\imath}\boldsymbol{\mathsf{x}}\llcorner} = b \boldsymbol{\mathsf{x}} \llcorner\urcorner = \underbrace{b \boldsymbol{\mathsf{x}} \urcorner}_{{}_{\imath}\boldsymbol{\mathsf{x}}\llcorner} \implies \text{LHS} = \underbrace{\llcorner\urcorner}_{{}_{\imath}\boldsymbol{\mathsf{x}}\llcorner} = \text{RHS}$$

$$\underbrace{1\mathbb{X}\urcorner}_{\lrcorner} = 1\mathbb{X}\urcorner$$

$$\underbrace{1\mathbb{X}\urcorner}_{\llcorner} = 1\mathbb{X}\urcorner$$

$$\overbrace{\underbrace{1\mathbb{X}\urcorner}\,\underbrace{\varphi\mathbb{X}\psi}}^{s:t} = \overbrace{\varphi\mathbb{X}\underbrace{\urcorner\psi}}^{s:t} = {}^{ts}\varphi\,\overbrace{{}^{t}\urcorner\psi} = {}^{ts}\varphi\,{}^{t}\urcorner\,{}^{t}\psi = {}^{t}\urcorner\,{}^{ts}\varphi\,{}^{t}\psi = \overbrace{1\mathbb{X}\urcorner}^{s:t}\,\overbrace{\varphi\mathbb{X}\psi}^{s:t} = \overbrace{1\mathbb{X}\urcorner\,\overbrace{\varphi\mathbb{X}\psi}}^{s:t}$$

$$\overbrace{\underbrace{1\mathbb{X}\urcorner}\,\underbrace{\varphi\mathbb{X}\psi}}^{s:t} = \overbrace{\varphi\mathbb{X}\underbrace{\urcorner\psi}}^{s:t} = {}^{st}\varphi\,\overbrace{{}^{t}\urcorner\psi} = {}^{st}\varphi\,{}^{t}\urcorner\,{}^{t}\psi = {}^{t}\urcorner\,{}^{st}\varphi\,{}^{t}\psi = \overbrace{1\mathbb{X}\urcorner}^{s:t}\,\overbrace{\varphi\mathbb{X}\psi}^{s:t} = \overbrace{\underbrace{1\mathbb{X}\urcorner}\,\overbrace{\varphi\mathbb{X}\psi}}^{s:t}$$

$$\begin{array}{ccc}
\underbrace{\mathbb{1}\mathbb{X}\mathbb{C}\overset{1}{\underset{\ell/r}{\diagdown}}\mathbb{h}}\;\mathbb{X}\;{}^{\mathbb{h}}_{\ell/r}\!\underset{0}{\diagup}\mathbb{C} & \ni & \begin{cases} \overbrace{\lrcorner\mathbb{1}\overbrace{\imath\mathbb{X}\overleftarrow{u}}^{\searrow}}\;\overbrace{\imath\mathbb{X}1\mathbb{X}\urcorner} \\ \overbrace{\llcorner\mathbb{1}\overbrace{\imath\mathbb{X}\vec{u}}^{\swarrow}}\;\overbrace{\imath\mathbb{X}1\mathbb{X}\urcorner} \end{cases} \\
\Big\downarrow \asymp & & \Big\downarrow \\
\mathbb{1}\mathbb{X}\Psi\big|\,{}^{\mathbb{h}}_{m}\!\underset{2}{\diagup}\mathbb{C} & \ni & \begin{cases} \lrcorner\mathbb{1}\overbrace{\imath\mathbb{X}\overleftarrow{u}\urcorner} \\ \llcorner\mathbb{1}\overbrace{\imath\mathbb{X}\vec{u}\urcorner} \end{cases}
\end{array}$$

$$\underbrace{\overbrace{\lrcorner\rrbracket\underbrace{\imath\bowtie\overleftarrow{u}}}^{\nsearrow}\overbrace{\imath\bowtie 1\bowtie\rceil}}_{\imath\bowtie\text{╝}} \;=\; {}_{\lrcorner\,\bowtie 1}\underbrace{\lrcorner\rrbracket\overbrace{\imath\bowtie\underbrace{\overleftarrow{u}}\rceil}}$$

$$\underbrace{\overbrace{\llcorner\rrbracket\underbrace{\imath\bowtie\overrightarrow{u}}}^{\nswarrow}\overbrace{\imath\bowtie 1\bowtie\rceil}}_{\imath\bowtie\text{╚}} \;=\; {}_{\llcorner\bowtie 1}\underbrace{\llcorner\rrbracket\overbrace{\imath\bowtie\underbrace{\overrightarrow{u}}\rceil}}$$

$$\text{LHS} = \underbrace{\overbrace{\lrcorner\rrbracket\bowtie 1}\overbrace{\imath\bowtie\overleftarrow{u}^{\urcorner}}\overbrace{\imath\bowtie 1\bowtie\rceil}}_{\imath\bowtie\text{╝}} = \underbrace{\lrcorner\rrbracket\bowtie 1}_{\imath\bowtie\text{╝}}\;\underbrace{\imath\bowtie\overleftarrow{u}^{\urcorner}}_{\imath\bowtie\text{╝}}\;\underbrace{\imath\bowtie 1\bowtie\rceil}_{\imath\bowtie\text{╝}} = {}_{\llcorner\bowtie 1}\underbrace{\lrcorner\rrbracket}\;\underbrace{\imath\bowtie 1\bowtie\overleftarrow{u}}\underbrace{\imath\bowtie 1\bowtie\rceil}$$

$$= {}_{\llcorner\bowtie 1}\underbrace{\lrcorner\rrbracket}\;\overbrace{\imath\bowtie 1\bowtie\underbrace{\overleftarrow{u}}\rceil} = {}_{\llcorner\bowtie 1}\underbrace{\lrcorner\rrbracket}\;\overbrace{\imath_{\llcorner}\bowtie\underbrace{\overleftarrow{u}}\rceil} = {}_{\llcorner\bowtie 1}\underbrace{\lrcorner\rrbracket}\;{}_{\llcorner\bowtie 1}\underbrace{\imath\bowtie\overbrace{\overleftarrow{u}}\rceil} = \text{RHS}$$

$$\text{LHS} = \underbrace{\overbrace{\llcorner\rrbracket\bowtie 1}\overbrace{\imath\bowtie\overrightarrow{u}^{\ulcorner}}\overbrace{\imath\bowtie 1\bowtie\rceil}}_{\imath\bowtie\text{╚}} = \underbrace{\llcorner\rrbracket\bowtie 1}_{\imath\bowtie\text{╚}}\;\underbrace{\imath\bowtie\overrightarrow{u}^{\ulcorner}}_{\imath\bowtie\text{╚}}\;\underbrace{\imath\bowtie 1\bowtie\rceil}_{\imath\bowtie\text{╚}} = {}_{\lrcorner\,\bowtie 1}\underbrace{\llcorner\rrbracket}\;\underbrace{\imath\bowtie 1\bowtie\overrightarrow{u}}\underbrace{\imath\bowtie 1\bowtie\rceil}$$

$$= {}_{\lrcorner\,\bowtie 1}\underbrace{\llcorner\rrbracket}\;\overbrace{\imath\bowtie 1\bowtie\underbrace{\overrightarrow{u}}\rceil} = {}_{\lrcorner\,\bowtie 1}\underbrace{\llcorner\rrbracket}\;\overbrace{\imath_{\lrcorner}\bowtie\underbrace{\overrightarrow{u}}\rceil} = {}_{\lrcorner\,\bowtie 1}\underbrace{\llcorner\rrbracket}\;{}_{\lrcorner\,\bowtie 1}\underbrace{\imath\bowtie\overbrace{\overrightarrow{u}}\rceil} = \text{RHS}$$