$$\overbrace{{}^{s:t}\urcorner\ \underbrace{\varphi\bowtie\psi}} = {}^{s}\varphi\ {}^{s^{-}t}\psi \Rightarrow \overbrace{\overleftarrow{g}\bowtie 1}^{\urcorner} = \overleftarrow{g}\bowtie\overleftarrow{g} \Rightarrow \overbrace{\overleftarrow{u}\bowtie 1}^{\urcorner} = \overleftarrow{u}^{\urcorner}$$

$$\overbrace{{}^{s:t}\ulcorner\ \underbrace{\varphi\bowtie\psi}} = {}^{s}\varphi\ {}^{ts^{-}}\psi \Rightarrow \overbrace{\overrightarrow{g}\bowtie\imath}^{\ulcorner} = \overrightarrow{g}\bowtie\overrightarrow{g} \Rightarrow \overbrace{\overrightarrow{u}\bowtie 1}^{\ulcorner} = \overrightarrow{u}^{\ulcorner}$$

$$\overbrace{{}^{s:t}\urcorner\ \underbrace{\underbrace{\overleftarrow{g}\bowtie 1}\underbrace{\varphi\bowtie\psi}}} = \overbrace{{}^{s:t}\urcorner\ \underbrace{\underbrace{\overleftarrow{g}\varphi}\bowtie\psi}} = {}^{s}\overbrace{\overleftarrow{g}\varphi}\ {}^{s^{-}t}\psi = {}^{g^{-}s}\varphi\ {}^{s^{-}t}\psi$$

$$= {}^{g^{-}s}\varphi\ {}^{\overline{\overbrace{g^{-}s}}\,g^{-}t}\psi = \overbrace{{}^{g^{-}s:g^{-}t}\urcorner\ \underbrace{\varphi\bowtie\psi}} = {}^{s:t}\overbrace{\underbrace{\overleftarrow{g}\bowtie\overleftarrow{g}}\ \overbrace{\urcorner\ \underbrace{\varphi\bowtie\psi}}}$$

$$\overbrace{{}^{s:t}\ulcorner\ \underbrace{\underbrace{\overrightarrow{g}\bowtie 1}\underbrace{\varphi\bowtie\psi}}} = \overbrace{{}^{s:t}\ulcorner\ \underbrace{\underbrace{\overrightarrow{g}\varphi}\bowtie\psi}} = {}^{s}\overbrace{\overrightarrow{g}\varphi}\ {}^{ts^{-}}\psi = {}^{sg}\varphi\ {}^{ts^{-}}\psi$$

$$= {}^{sg}\varphi\ {}^{tg(sg)^{-}}\psi = \overbrace{{}^{sg:tg}\ulcorner\ \underbrace{\varphi\bowtie\psi}} = {}^{s:t}\overbrace{\underbrace{\overrightarrow{g}\bowtie\overrightarrow{g}}\ \overbrace{\ulcorner\ \underbrace{\varphi\bowtie\psi}}}$$

$$x \in \mathbb{1}\bowtie\mathbb{C}\overset{1}{\underset{\ell/r}{\triangleleft}}\hbar \xrightarrow[\begin{cases}\text{left coaction}\\ \text{right coaction}\end{cases}]{\begin{cases}\bullet\lrcorner\\ \not\curlyvee\end{cases}} \underbrace{\mathbb{1}\bowtie\mathbb{C}\overset{1}{\underset{\ell/r}{\triangleleft}}\hbar}\bowtie\mathbb{C}\overset{1}{\underset{\ell/r}{\triangleleft}}\hbar \ni \begin{cases} x^{\curlyvee} = \overbrace{x\bowtie 1}^{\imath\bowtie\urcorner} \\ x^{\not\curlyvee} = \overbrace{x\bowtie 1}^{\imath\bowtie\ulcorner}\end{cases}$$

$$\overbrace{\imath\bowtie 1}^{\urcorner} = \imath\bowtie 1$$

$$\overbrace{\imath\bowtie 1}^{\ulcorner} = \imath\bowtie 1$$

$$\overbrace{{}^{s:t}\urcorner\ \underbrace{\underbrace{\imath\bowtie 1}\underbrace{\varphi\bowtie\psi}}} = \overbrace{{}^{s:t}\urcorner\ \underbrace{\underbrace{\imath\varphi}\bowtie\psi}} = {}^{s}\overbrace{\imath\varphi}\ {}^{s^{-}t}\psi$$

$$= {}^{s}\imath\ {}^{s}\varphi\ {}^{s^{-}t}\psi = {}^{s:t}\overbrace{\imath\bowtie 1}\ \overbrace{{}^{s:t}\urcorner\ \underbrace{\varphi\bowtie\psi}} = {}^{s:t}\overbrace{\underbrace{\imath\bowtie 1}\ \overbrace{\urcorner\ \underbrace{\varphi\bowtie\psi}}}$$

$$\overbrace{{}^{s:t}\ulcorner\ \underbrace{\underbrace{\imath\bowtie 1}\underbrace{\varphi\bowtie\psi}}} = \overbrace{{}^{s:t}\ulcorner\ \underbrace{\underbrace{\imath\varphi}\bowtie\psi}} = {}^{s}\overbrace{\imath\varphi}\ {}^{ts^{-}}\psi$$

$$= {}^{s}\imath\ {}^{s}\varphi\ {}^{ts^{-}}\psi = {}^{s:t}\overbrace{\imath\bowtie 1}\ \overbrace{{}^{s:t}\ulcorner\ \underbrace{\varphi\bowtie\psi}} = {}^{s:t}\overbrace{\underbrace{\imath\bowtie 1}\ \overbrace{\ulcorner\ \underbrace{\varphi\bowtie\psi}}}$$

$$\overbrace{\lrcorner\urcorner\mathbb{1}}^{\imath\mathbb{1}\urcorner} = \lrcorner\urcorner\mathbb{1}$$

$$\overbrace{\llcorner\urcorner\mathbb{1}}^{\imath\mathbb{1}\urcorner} = \llcorner\urcorner\mathbb{1}$$

$$\mathbb{1} \mathbb{1} \overset{\urcorner}{\underset{0}{\triangleleft}} \mathbb{C} \ni \lrcorner\urcorner = b_i \mathbb{1} \urcorner^i \implies \overbrace{b\mathbb{1}\urcorner\mathbb{1}}^{\imath\mathbb{1}\urcorner} = b\mathbb{1}\overbrace{\urcorner\mathbb{1}}^{\urcorner} = b\mathbb{1}\urcorner\mathbb{1}$$

$$\mathbb{1} \mathbb{1} \overset{\urcorner}{\underset{0}{\triangleleft}} \mathbb{C} \ni \llcorner\urcorner = b_i \mathbb{1} \urcorner^i \implies \overbrace{b\mathbb{1}\urcorner\mathbb{1}}^{\imath\mathbb{1}\urcorner} = b\mathbb{1}\overbrace{\urcorner\mathbb{1}}^{\urcorner} = b\mathbb{1}\urcorner\mathbb{1}$$

$$\overbrace{\lrcorner\urcorner\imath\mathbb{1}\overleftarrow{u}}^{\swarrow} = \overbrace{\lrcorner\urcorner\mathbb{1}}\,\underbrace{\imath\mathbb{1}\overleftarrow{u}\urcorner}$$

$$\overbrace{\llcorner\urcorner\imath\mathbb{1}\overrightarrow{u}}^{\searrow} = \overbrace{\llcorner\urcorner\mathbb{1}}\,\underbrace{\imath\mathbb{1}\overrightarrow{u}\lrcorner}$$

$$\text{LHS} = \overbrace{\underbrace{\lrcorner\urcorner\overbrace{\imath\mathbb{1}\overleftarrow{u}}^{\imath\mathbb{1}\urcorner}}\mathbb{1}}^{\imath\mathbb{1}\urcorner} = \overbrace{\underbrace{\lrcorner\urcorner\mathbb{1}}\underbrace{\imath\mathbb{1}\overleftarrow{u}\mathbb{1}}}^{\imath\mathbb{1}\urcorner} = \overbrace{\lrcorner\urcorner\mathbb{1}}^{\imath\mathbb{1}\urcorner}\overbrace{\imath\mathbb{1}\overleftarrow{u}\mathbb{1}}^{\imath\mathbb{1}\urcorner} = \overbrace{\lrcorner\urcorner\mathbb{1}}^{\imath\mathbb{1}\urcorner}\underbrace{\imath\mathbb{1}\overbrace{\overleftarrow{u}\mathbb{1}}^{\urcorner}} = \text{RHS}$$

$$\text{LHS} = \overbrace{\underbrace{\llcorner\urcorner\overbrace{\imath\mathbb{1}\overrightarrow{u}}^{\imath\mathbb{1}\lrcorner}}\mathbb{1}}^{\imath\mathbb{1}\lrcorner} = \overbrace{\underbrace{\llcorner\urcorner\mathbb{1}}\underbrace{\imath\mathbb{1}\overrightarrow{u}\mathbb{1}}}^{\imath\mathbb{1}\lrcorner} = \overbrace{\llcorner\urcorner\mathbb{1}}^{\imath\mathbb{1}\lrcorner}\overbrace{\imath\mathbb{1}\overrightarrow{u}\mathbb{1}}^{\imath\mathbb{1}\lrcorner} = \overbrace{\llcorner\urcorner\mathbb{1}}^{\imath\mathbb{1}\lrcorner}\underbrace{\imath\mathbb{1}\overbrace{\overrightarrow{u}\mathbb{1}}^{\lrcorner}} = \text{RHS}$$