

$$\begin{aligned} s:t \overline{\overline{\varphi} \mathbf{x} \psi}^{\lrcorner} &= {}^s\varphi \, {}^{s^{-t}}\psi \Rightarrow \overline{\overline{g} \mathbf{x} 1}^{\lrcorner} = \overline{g} \, \mathbf{x} \, \overline{g} \Rightarrow \overline{\overline{u} \mathbf{x} 1}^{\lrcorner} = \overline{u}^{\lrcorner} \\ s:t \overline{\overline{\varphi} \mathbf{x} \psi}^{\ulcorner} &= {}^s\varphi \, {}^{ts^{-}}\psi \Rightarrow \overline{\overline{g} \mathbf{x} i}^{\ulcorner} = \overline{g} \, \mathbf{x} \, \overline{g} \Rightarrow \overline{\overline{u} \mathbf{x} 1}^{\ulcorner} = \overline{u}^{\ulcorner} \end{aligned}$$

$$\begin{aligned} s:t \overline{\overline{\overline{g} \mathbf{x} 1} \varphi \mathbf{x} \psi}^{\lrcorner} &= s:t \overline{\overline{\overline{g} \varphi} \mathbf{x} \psi}^{\lrcorner} = \overline{{}^s\overline{g} \varphi}^{\lrcorner} \, {}^{s^{-t}}\psi = {}^{g^{-}s} \varphi \, {}^{s^{-t}}\psi \\ &= {}^{g^{-}s} \varphi \, \overline{\overline{g^{-}s} g^{-t}} \psi = {}^{g^{-}s: g^{-t}} \overline{\overline{\varphi} \mathbf{x} \psi}^{\lrcorner} = \overline{\overline{\overline{g} \mathbf{x} \overline{g}} \overline{\overline{\varphi} \mathbf{x} \psi}}^{\lrcorner} \\ s:t \overline{\overline{\overline{g} \mathbf{x} 1} \varphi \mathbf{x} \psi}^{\ulcorner} &= s:t \overline{\overline{\overline{g} \varphi} \mathbf{x} \psi}^{\ulcorner} = \overline{{}^s\overline{g} \varphi}^{\ulcorner} \, {}^{ts^{-}}\psi = {}^{sg} \varphi \, {}^{ts^{-}}\psi \\ &= {}^{sg} \varphi \, {}^{tg(sg)^{-}} \psi = {}^{sg:tg} \overline{\overline{\varphi} \mathbf{x} \psi}^{\ulcorner} = \overline{\overline{\overline{g} \mathbf{x} \overline{g}} \overline{\overline{\varphi} \mathbf{x} \psi}}^{\ulcorner} \end{aligned}$$

$$x \in \mathbb{1} \mathbf{x} \mathbb{C} \mathbb{1}_{\ell/r} \xrightarrow{\begin{cases} \vdash \\ \not\vdash \\ \text{left coaction} \\ \text{right coaction} \end{cases}} \underbrace{\mathbb{1} \mathbf{x} \mathbb{C} \mathbb{1}_{\ell/r}}_{\mathbf{x} \mathbb{C} \mathbb{1}_{\ell/r}} \ni \begin{cases} x^{\lrcorner} = \overline{\overline{x} \mathbf{x} 1}^{\lrcorner} \\ x^{\ulcorner} = \overline{\overline{x} \mathbf{x} 1}^{\ulcorner} \end{cases}$$

$$\begin{aligned} \overline{\overline{\gamma} \mathbf{x} 1}^{\lrcorner} &= \gamma \mathbf{x} 1 \\ \overline{\overline{\gamma} \mathbf{x} 1}^{\ulcorner} &= \gamma \mathbf{x} 1 \end{aligned}$$

$$\begin{aligned} s:t \overline{\overline{\overline{\gamma} \mathbf{x} 1} \varphi \mathbf{x} \psi}^{\lrcorner} &= s:t \overline{\overline{\overline{\gamma} \varphi} \mathbf{x} \psi}^{\lrcorner} = \overline{{}^s\overline{\gamma} \varphi}^{\lrcorner} \, {}^{s^{-t}}\psi \\ &= {}^s\gamma \, {}^s\varphi \, {}^{s^{-t}}\psi = {}^{s:t} \overline{\overline{\gamma} \mathbf{x} 1}^{\lrcorner} \, {}^{s:t} \overline{\overline{\varphi} \mathbf{x} \psi}^{\lrcorner} = \overline{\overline{\overline{\gamma} \mathbf{x} 1} \overline{\overline{\varphi} \mathbf{x} \psi}}^{\lrcorner} \\ s:t \overline{\overline{\overline{\gamma} \mathbf{x} 1} \varphi \mathbf{x} \psi}^{\ulcorner} &= s:t \overline{\overline{\overline{\gamma} \varphi} \mathbf{x} \psi}^{\ulcorner} = \overline{{}^s\overline{\gamma} \varphi}^{\ulcorner} \, {}^{ts^{-}}\psi \\ &= {}^s\gamma \, {}^s\varphi \, {}^{ts^{-}}\psi = {}^{s:t} \overline{\overline{\gamma} \mathbf{x} 1}^{\ulcorner} \, {}^{s:t} \overline{\overline{\varphi} \mathbf{x} \psi}^{\ulcorner} = \overline{\overline{\overline{\gamma} \mathbf{x} 1} \overline{\overline{\varphi} \mathbf{x} \psi}}^{\ulcorner} \end{aligned}$$

$$\overline{\lrcorner \mathbb{I} \mathbf{x} 1}^{\imath \mathbf{x} \neg} = \lrcorner \mathbb{I} \mathbf{x} 1$$

$$\overline{\lrcorner \mathbb{I} \mathbf{x} 1}^{\imath \mathbf{x} \ulcorner} = \lrcorner \mathbb{I} \mathbf{x} 1$$

$$\mathbb{I} \mathbf{x} \begin{smallmatrix} \mathbb{H} \\ \triangle_0 \end{smallmatrix} C \ni \lrcorner \mathbb{I} = b_i \mathbf{x} \dot{\neg} \Rightarrow \overline{b \mathbf{x} \neg \mathbf{x} 1}^{\imath \mathbf{x} \neg} = b \mathbf{x} \overline{\neg \mathbf{x} 1}^{\neg} = b \mathbf{x} \neg \mathbf{x} 1$$

$$\mathbb{I} \mathbf{x} \begin{smallmatrix} \mathbb{H} \\ \triangle_0 \end{smallmatrix} C \ni \lrcorner \mathbb{I} = b_i \mathbf{x} \dot{\neg} \Rightarrow \overline{b \mathbf{x} \neg \mathbf{x} 1}^{\imath \mathbf{x} \ulcorner} = b \mathbf{x} \overline{\neg \mathbf{x} 1}^{\ulcorner} = b \mathbf{x} \neg \mathbf{x} 1$$

$$\overline{\lrcorner \mathbb{I} \overline{\imath \mathbf{x} u}}^{\neg} = \overline{\lrcorner \mathbb{I} \mathbf{x} 1}^{\neg} \overline{\imath \mathbf{x} u}^{\neg}$$

$$\overline{\lrcorner \mathbb{I} \overrightarrow{\imath \mathbf{x} u}}^{\ulcorner} = \overline{\lrcorner \mathbb{I} \mathbf{x} 1}^{\ulcorner} \overline{\imath \mathbf{x} u}^{\ulcorner}$$

$$\text{LHS} = \overline{\lrcorner \mathbb{I} \overline{\imath \mathbf{x} u}^{\neg} \mathbf{x} 1}^{\imath \mathbf{x} \neg} = \overline{\lrcorner \mathbb{I} \mathbf{x} 1}^{\imath \mathbf{x} \neg} \overline{\imath \mathbf{x} u^{\neg} \mathbf{x} 1}^{\imath \mathbf{x} \neg} = \overline{\lrcorner \mathbb{I} \mathbf{x} 1}^{\imath \mathbf{x} \neg} \overline{\imath \mathbf{x} u^{\neg} \mathbf{x} 1}^{\neg} = \overline{\lrcorner \mathbb{I} \mathbf{x} 1}^{\neg} \overline{\imath \mathbf{x} u^{\neg} \mathbf{x} 1}^{\neg} = \text{RHS}$$

$$\text{LHS} = \overline{\lrcorner \mathbb{I} \overrightarrow{\imath \mathbf{x} u}^{\ulcorner} \mathbf{x} 1}^{\imath \mathbf{x} \ulcorner} = \overline{\lrcorner \mathbb{I} \mathbf{x} 1}^{\imath \mathbf{x} \ulcorner} \overline{\imath \mathbf{x} u^{\ulcorner} \mathbf{x} 1}^{\imath \mathbf{x} \ulcorner} = \overline{\lrcorner \mathbb{I} \mathbf{x} 1}^{\imath \mathbf{x} \ulcorner} \overline{\imath \mathbf{x} u^{\ulcorner} \mathbf{x} 1}^{\ulcorner} = \overline{\lrcorner \mathbb{I} \mathbf{x} 1}^{\ulcorner} \overline{\imath \mathbf{x} u^{\ulcorner} \mathbf{x} 1}^{\ulcorner} = \text{RHS}$$

$$\begin{array}{ccc} \mathbb{I} \mathbf{x} C_{\ell/r} \begin{smallmatrix} \neg \\ \mathbb{H} \end{smallmatrix} & \xrightarrow{\left\{ \begin{array}{l} \neg \\ \ulcorner \end{array} \right\}} & \overline{\mathbb{I} \mathbf{x} C_{\ell/r} \begin{smallmatrix} \neg \\ \mathbb{H} \end{smallmatrix} \mathbf{x} C_{\ell/r} \begin{smallmatrix} \neg \\ \mathbb{H} \end{smallmatrix}} \\ \downarrow \left\{ \begin{array}{l} \neg \\ \ulcorner \end{array} \right\} & & \downarrow \left\{ \begin{array}{l} \neg \mathbf{x} 1 \\ \ulcorner \mathbf{x} 1 \end{array} \right\} \\ \overline{\mathbb{I} \mathbf{x} C_{\ell/r} \begin{smallmatrix} \neg \\ \mathbb{H} \end{smallmatrix} \mathbf{x} C_{\ell/r} \begin{smallmatrix} \neg \\ \mathbb{H} \end{smallmatrix}} & \xrightarrow{\left\{ \begin{array}{l} \imath \mathbf{x} 1 \mathbf{x} \neg \\ \imath \mathbf{x} 1 \mathbf{x} \ulcorner \end{array} \right\}} & \overline{\mathbb{I} \mathbf{x} C_{\ell/r} \begin{smallmatrix} \neg \\ \mathbb{H} \end{smallmatrix} \mathbf{x} C_{\ell/r} \begin{smallmatrix} \neg \\ \mathbb{H} \end{smallmatrix} \mathbf{x} C_{\ell/r} \begin{smallmatrix} \neg \\ \mathbb{H} \end{smallmatrix}} \end{array}$$