

$$\begin{aligned}
\overrightarrow{s:t \varphi \mathbf{x} \psi} &= {}^{st^-} \varphi {}^t \psi \\
\overleftarrow{s:t \varphi \mathbf{x} \psi} &= {}^{t^-} s \varphi {}^t \psi \\
\overrightarrow{st} \lrcorner &= {}^{st^-} \lrcorner \\
\overleftarrow{st} \lrcorner &= {}^{t^-} s \lrcorner
\end{aligned}$$

$$\begin{aligned}
\lrcorner \mathbf{x} 1 &= \rightarrow \lrcorner \text{ bi-mult} \\
\downarrow \\
\lrcorner \mathbf{x} 1 &= \leftarrow \lrcorner \text{ bi-mult} \\
\downarrow
\end{aligned}$$

$$\begin{aligned}
\overrightarrow{s:t \lrcorner \mathbf{x} 1 \varphi \mathbf{x} \psi} &= \overrightarrow{s:t \lrcorner \varphi \mathbf{x} \psi} = {}^{st^-} \lrcorner \varphi {}^t \psi = {}^{st^-} \lrcorner {}^{st^-} \varphi {}^t \psi = \overrightarrow{s:t} \lrcorner \overrightarrow{s:t \varphi \mathbf{x} \psi} = \overrightarrow{s:t \rightarrow \lrcorner \varphi \mathbf{x} \psi} \\
\overleftarrow{s:t \lrcorner \mathbf{x} 1 \varphi \mathbf{x} \psi} &= \overleftarrow{s:t \lrcorner \varphi \mathbf{x} \psi} = {}^{t^-} s \lrcorner \varphi {}^t \psi = {}^{t^-} s \lrcorner {}^{t^-} s \varphi {}^t \psi = \overleftarrow{s:t} \lrcorner \overleftarrow{s:t \varphi \mathbf{x} \psi} = \overleftarrow{s:t \leftarrow \lrcorner \varphi \mathbf{x} \psi}
\end{aligned}$$

$$\left\{ \begin{array}{l} \nearrow x = \frac{x \mathbf{x} 1}{\imath \mathbf{x} \downarrow} \\ \nwarrow x = \frac{x \mathbf{x} 1}{\imath \mathbf{x} \downarrow} \end{array} \right. \in \underbrace{\mathbb{I} \mathbf{x} \frac{\mathbb{H} \omega}{\ell/r \ 0} \mathbb{C} \mathbf{x} \frac{\mathbb{H} \omega}{0} \mathbb{C}}_{\substack{\text{left cooption} \\ \text{right cooption}}} \xleftarrow{\begin{Bmatrix} \nearrow \\ \nwarrow \end{Bmatrix}} \mathbb{I} \mathbf{x} \frac{\mathbb{H} \omega}{\ell/r \ 0} \mathbb{C} \ni x$$

$$\begin{aligned}
\overrightarrow{\underline{g} \mathbf{x} 1} &= \overrightarrow{\underline{g} \mathbf{x} 1} \Rightarrow \overrightarrow{\underline{u} \mathbf{x} 1} = \overrightarrow{\underline{u} \mathbf{x} 1} \\
\downarrow & \qquad \qquad \downarrow \\
\overrightarrow{\underline{g} \mathbf{x} 1} &= \overrightarrow{\underline{g} \mathbf{x} 1} \Rightarrow \overrightarrow{\underline{u} \mathbf{x} 1} = \overrightarrow{\underline{u} \mathbf{x} 1} \\
\downarrow & \qquad \qquad \downarrow
\end{aligned}$$

$$\begin{aligned}
\overrightarrow{s:t \overrightarrow{\underline{g} \mathbf{x} 1} \varphi \mathbf{x} \psi} &= \overrightarrow{s:t \overrightarrow{\underline{g} \varphi \mathbf{x} \psi}} = {}^{st^-} \overrightarrow{\underline{g} \varphi} {}^t \psi = {}^{g^- st^-} \varphi {}^t \psi = \overrightarrow{s:t} \varphi \mathbf{x} \psi = \overrightarrow{s:t \overrightarrow{\underline{g} \mathbf{x} 1} \varphi \mathbf{x} \psi} \\
\overleftarrow{s:t \overrightarrow{\underline{g} \mathbf{x} 1} \varphi \mathbf{x} \psi} &= \overleftarrow{s:t \overrightarrow{\underline{g} \varphi \mathbf{x} \psi}} = {}^{t^-} s \overrightarrow{\underline{g} \varphi} {}^t \psi = {}^{t^- sg} \varphi {}^t \psi = \overleftarrow{s:t} \varphi \mathbf{x} \psi = \overleftarrow{s:t \overrightarrow{\underline{g} \mathbf{x} 1} \varphi \mathbf{x} \psi}
\end{aligned}$$

$$\underbrace{\mathbb{L}^{\lrcorner} \mathbf{x} 1}_{i \mathbf{x} \Downarrow} = \mathbb{L}^{\lrcorner} \mathbf{x} 1$$

$$\underbrace{\mathbb{L}^{\lceil} \mathbf{x} 1}_{i \mathbf{x} \lceil} = \mathbb{L}^{\lceil} \mathbf{x} 1$$

$$\mathbb{T}^{\omega} \mathbf{x} \mathbb{C} \bigtriangleup_{\ell}^{\lceil} \mathfrak{h} \ni \mathbb{L}^{\lrcorner} = b_i \mathbf{x} \overleftarrow{u}^i: \quad \underbrace{b \mathbf{x} \overleftarrow{u} \mathbf{x} 1}_{i \mathbf{x} \Downarrow} = b \mathbf{x} \underbrace{\overleftarrow{u} \mathbf{x} 1}_{\Downarrow} = b \mathbf{x} \overleftarrow{u} \mathbf{x} 1$$

$$\mathbb{T}^{\omega} \mathbf{x} \mathbb{C} \bigtriangleup_r^{\lceil} \mathfrak{h} \ni \mathbb{L}^{\lceil} = b_i \mathbf{x} \overrightarrow{u}^i: \quad \underbrace{b \mathbf{x} \overrightarrow{u} \mathbf{x} 1}_{i \mathbf{x} \lceil} = b \mathbf{x} \underbrace{\overrightarrow{u} \mathbf{x} 1}_{\lceil} = b \mathbf{x} \overrightarrow{u} \mathbf{x} 1$$

$$\begin{array}{c} \nearrow \\ \underbrace{\mathbb{L}^{\lceil} \overbrace{i\mathbf{x}\rceil}} = \underbrace{\mathbb{L}^{\lceil} \mathbf{x}1}_{i\mathbf{x}\downarrow} \overbrace{i\mathbf{x}\rceil} \\ \nwarrow \\ \underbrace{\mathbb{L}^{\lceil} \overbrace{i\mathbf{x}\rceil}} = \underbrace{\mathbb{L}^{\lceil} \mathbf{x}1}_{i\mathbf{x}\downarrow} \overbrace{i\mathbf{x}\nwarrow} \end{array}$$

$$\underbrace{i\mathbf{x}\rceil\mathbf{x}1}_{i\mathbf{x}\downarrow} = i\mathbf{x}\rceil$$

$$\underbrace{i\mathbf{x}\rceil\mathbf{x}1}_{i\mathbf{x}\downarrow} = i\mathbf{x}\nwarrow$$

$$t \bowtie x := \overset{t^-}{\nearrow} x = i\mathbf{x}\downarrow x$$

$$\overset{t^-}{\nearrow} \underbrace{\mathbb{L}^{\lceil} \overbrace{i\mathbf{x}\rceil}} = \overset{t^-}{\underbrace{\mathbb{L}^{\lceil} \mathbf{x}1}_{i\mathbf{x}\downarrow} \overbrace{i\mathbf{x}\rceil}} = \overset{t^-}{\underbrace{\mathbb{L}^{\lceil} \mathbf{x}1}_{i\mathbf{x}\downarrow} \overset{t^-}{\underbrace{\overbrace{i\mathbf{x}\rceil}}}} = \mathbb{L}^{\lceil} \overbrace{i\mathbf{x}\downarrow} = \underbrace{\mathbb{L}^{\lceil} \overbrace{i\mathbf{x}\rceil}}_{i\mathbf{x}\downarrow} = \underbrace{\mathbb{L}^{\lceil} \overbrace{i\mathbf{x}\rceil}}_{i\mathbf{x}\downarrow}$$

$$\text{LHS} = \underbrace{\underbrace{\mathbb{L}^{\lceil} \overbrace{i\mathbf{x}\rceil}}_{i\mathbf{x}\downarrow} \mathbf{x}1}_{i\mathbf{x}\downarrow} = \underbrace{\underbrace{\mathbb{L}^{\lceil} \mathbf{x}1}_{i\mathbf{x}\downarrow} \overbrace{i\mathbf{x}\rceil\mathbf{x}1}}_{i\mathbf{x}\downarrow} = \underbrace{\mathbb{L}^{\lceil} \mathbf{x}1}_{i\mathbf{x}\downarrow} \underbrace{\overbrace{i\mathbf{x}\rceil\mathbf{x}1}_{i\mathbf{x}\downarrow}} = \text{RHS}$$

$$t \bowtie x = \nwarrow x = i\mathbf{x}\downarrow x$$

$$\nwarrow \underbrace{\mathbb{L}^{\lceil} \overbrace{i\mathbf{x}\rceil}} = \underbrace{\mathbb{L}^{\lceil} \mathbf{x}1}_{i\mathbf{x}\downarrow} \overbrace{i\mathbf{x}\rceil} = \underbrace{\mathbb{L}^{\lceil} \mathbf{x}1}_{i\mathbf{x}\downarrow} \overset{t}{\underbrace{\overbrace{i\mathbf{x}\rceil}}} = \mathbb{L}^{\lceil} \overbrace{i\mathbf{x}\downarrow} = \underbrace{\mathbb{L}^{\lceil} \overbrace{i\mathbf{x}\rceil}}_{i\mathbf{x}\downarrow} = \underbrace{\mathbb{L}^{\lceil} \overbrace{i\mathbf{x}\rceil}}_{i\mathbf{x}\downarrow}$$

$$\text{LHS} = \underbrace{\underbrace{\mathbb{L}^{\lceil} \overbrace{i\mathbf{x}\rceil}}_{i\mathbf{x}\downarrow} \mathbf{x}1}_{i\mathbf{x}\downarrow} = \underbrace{\underbrace{\mathbb{L}^{\lceil} \mathbf{x}1}_{i\mathbf{x}\downarrow} \overbrace{i\mathbf{x}\rceil\mathbf{x}1}}_{i\mathbf{x}\downarrow} = \underbrace{\mathbb{L}^{\lceil} \mathbf{x}1}_{i\mathbf{x}\downarrow} \underbrace{\overbrace{i\mathbf{x}\rceil\mathbf{x}1}_{i\mathbf{x}\downarrow}} = \text{RHS}$$

$$\begin{array}{ccc} \underbrace{\mathbb{L}^{\omega} \overline{\mathbb{H}}_{\ell/r}^{\omega} \mathbb{C} \mathbf{x} \overline{\mathbb{H}}_0^{\omega} \mathbb{C}} & \xleftarrow{\begin{Bmatrix} \nearrow \\ \nwarrow \end{Bmatrix}} & \mathbb{L}^{\omega} \overline{\mathbb{H}}_{\ell/r}^{\omega} \mathbb{C} \\ \downarrow \begin{Bmatrix} \nearrow \mathbf{x}1 \\ \nwarrow \mathbf{x}1 \end{Bmatrix} & & \downarrow \begin{Bmatrix} \nearrow \\ \nwarrow \end{Bmatrix} \\ \underbrace{\mathbb{L}^{\omega} \overline{\mathbb{H}}_{\ell/r}^{\omega} \mathbb{C} \mathbf{x} \overline{\mathbb{H}}_0^{\omega} \mathbb{C} \mathbf{x} \overline{\mathbb{H}}_0^{\omega} \mathbb{C}} & \xleftarrow{\begin{Bmatrix} i\mathbf{x}\bowtie \\ i\mathbf{x}\bowtie \end{Bmatrix}} & \underbrace{\mathbb{L}^{\omega} \overline{\mathbb{H}}_{\ell/r}^{\omega} \mathbb{C} \mathbf{x} \overline{\mathbb{H}}_0^{\omega} \mathbb{C}} \end{array}$$