

$$\mathcal{H}_{\pm}=\left\{\begin{array}{l}\mathcal{H}_{+} \\ \mathcal{H}_{-}\end{array}\right.$$

$$\mathcal{H}^{\pm}=\left\{\begin{array}{l}\mathcal{H}^{+} \\ \mathcal{H}^{-}\end{array}\right.$$

$$\psi\left(t+2\pi\right)=\pm\psi\left(t\right)$$

$$\text{closed string Hilbert space}$$

$$\mathcal{H}=\mathcal{H}_{\pm}\boxtimes\mathcal{H}^{\pm}=\left\{\begin{array}{l}\mathcal{H}_{+} \\ \mathcal{H}_{-}\end{array}\right.\boxtimes\left\{\begin{array}{l}\mathcal{H}^{+} \\ \mathcal{H}^{-}\end{array}\right.=\left\{\begin{array}{l}\mathcal{H}_{+}\boxtimes\mathcal{H}^{+} \\ \mathcal{H}_{-}\boxtimes\mathcal{H}^{-} \\ \mathcal{H}_{+}\boxtimes\mathcal{H}^{-} \\ \mathcal{H}_{-}\boxtimes\mathcal{H}^{+}\end{array}\right.=\left\{\begin{array}{l}\mathcal{H}_{+}^{+} \\ \mathcal{H}_{-}^{-}\end{array}\right.\times\left\{\begin{array}{l}\mathcal{H}_{+}^{-} \\ \mathcal{H}_{-}^{+}\end{array}\right.$$