$$H_2\left(\mathbb{P}^2_k{:}\mathbb{Z}\right) = \mathbb{Z}^{1:k} = \mathbb{Z}K \times \mathbb{K}^{\perp}$$

$$K^{\perp} = E_k \text{ root lattice}$$

$$\text{rat}^0 = \text{ roots } = \text{ BPS instantons } \begin{cases} A|A = -2 \\ K|A = 0 \end{cases}$$

$$s_A(C) = C + \underbrace{A|C}A$$

$$s_A^2 = \text{ id}$$

$$s_A^2(C) = s_A(C) + \underbrace{A|s_A(C)}A = C + \underbrace{A|C}A + \underbrace{A|\overbrace{C + \underbrace{A|C}A}}A = C + \underbrace{A|C}A + \underbrace{A|C}A + \underbrace{A|C}\underbrace{A|A}_{=-2}A = C$$

$$s_A(C)\,|\,s_A\left(C'\right) = C|C'$$

$$\text{LHS } = \overbrace{C + \underbrace{A|C}A}|\overbrace{C' + \underbrace{A|C'}A} = C|C' + \underbrace{A|C}\underbrace{A|C'} + \underbrace{A|C'}\underbrace{C|A} + \underbrace{A|C}\underbrace{A|C'}\underbrace{A|A}_{=-2} = \text{ RHS}$$

$$s_A(K) = K \Longrightarrow s_A\left(K^{\perp}\right) = K^{\perp}$$

$$\text{roots } \begin{cases} C_i - C_j \\ C_0 - C_i - C_j - C_\ell \end{cases}$$

$$\underbrace{C_i - C_j}|\underbrace{C_i - C_j} = C_i|C_i + C_j|C_j = -1 - 1 = -2$$

$$-K|\underbrace{C_i - C_j} = \underbrace{3C_0 - C_1 - \cdots - C_k}|\underbrace{C_i - C_j} = -C_i|C_i + C_j|C_j = 1 - 1 = 0$$

$$\underbrace{C_0 - C_i - C_j - C_\ell}|\underbrace{C_0 - C_i - C_j - C_\ell} = C_0|C_0 + C_i|C_i + C_j|C_j + C_\ell|C_\ell = 1 - 1 - 1 - 1 = -2$$

$$-K|\underbrace{C_0 - C_i - C_j - C_\ell} = \underbrace{3C_0 - C_1 - \cdots - C_k}|\underbrace{C_0 - C_i - C_j - C_\ell} = 3\,C_0|C_0 + C_i|C_i + C_j|C_j + C_\ell|C_\ell = 3 - 1 -$$

$$\text{simple roots } \begin{cases} A_i = C_i - C_{i+1} & 1 \leqslant i < k \\ A_k = C_0 - C_1 - C_2 - C_3 \end{cases}$$

$$\text{Pezzo } -K = c_1(X) > 0 \text{ pos scalar curvature}$$
$$c_1^2(X) = K|K = 9 - k > 0 \Longrightarrow k \leqslant 8$$