IIA massless branes

9+branes

$$1\text{+brane} \int^{\Sigma^2} B_{\mu\nu}\, \varepsilon^{\alpha\beta}\, \underbrace{\partial_{\alpha\,0} 1\!\!\mathbb{L}^{\mu}}\, \underbrace{\partial_{\beta\,0} 1\!\!\mathbb{L}^{\nu}}$$

$$\frac{1}{2}\ \text{BPS states}: \quad \not{4}: \quad \begin{matrix}1\\5\end{matrix}\vdots \quad \begin{matrix}0\\6\end{matrix}\vdots \quad \begin{matrix}2\\4\end{matrix}\vdots \quad \begin{matrix}-2\\8\end{matrix}$$

$$\not{4}\begin{matrix}1\\5\end{matrix} = \not{4}\begin{matrix}2\\5\end{matrix} \boxminus \begin{matrix}1\\0\end{matrix}\vdots \quad \not{4}\begin{matrix}2\\4\end{matrix} = \not{4}\begin{matrix}2\\5\end{matrix} \boxminus \begin{matrix}0\\1\end{matrix}\vdots \quad \not{4}\begin{matrix}0\\6\end{matrix}$$

$$k = 1A$$

$$\text{Pic}\ \mathbb{P}^2_1 = \mathbb{Z} < C_0 : C_1 >$$

$$-K = 3C_0 - C_1\ \text{ample}$$

$$\text{rational}\ C|\,(K + C) = -2$$

$$g_C = 1 + \frac{C|\,(K + C)}{2} = 0$$

$$\text{degree}\ -K|C$$

$$\text{Serre/em dual}\ \widetilde{C} = -C - K$$

$$C\ \text{rat} \Leftrightarrow \widetilde{C}\ \text{rat}$$

$$\deg C + \deg \widetilde{C} = -K|C + -K|\,(-C - K) = -K| - K = 9 - 1 = 8$$

min pos roots

$$\text{genus 0 min pos deg} \begin{cases}\alpha_0 = C_0 - C_1 = 1| - 1\\ \alpha_1 = C_1 = 0|1\end{cases}$$

pos roots

deg=0 K orthogonal Serre/em dual

rational holomorphic = BPS /2

$$\begin{cases}0 & 0\\ 3\alpha_0 + 2\alpha_1 = 3C_0 - C_1 = -K & 8\end{cases} \begin{cases}\not{0}\\ \hline *\underline{\not{0}}\end{cases}$$

$$\not{4}\frac{1}{5} = \not{4}\frac{2}{5} \boxminus \frac{1}{0} = {}^{2}_{6}\mathrm{rat} \begin{cases} \alpha_0 = C_0 - C_1 & \bcancel{2} \sim \cancel{2} \\ 2\alpha_0 + 2\alpha_1 = 2C_0 & \bcancel{6} \sim \overline{*\cancel{2}}_{-} \end{cases} \begin{cases} M_2/1 = F_1 = \dfrac{1}{\ell_s^2} \sim 1/g_s^0 \\ M_5/0 = N_5 = \dfrac{1}{g_s^2 \ell_s^6} \sim 1/g_s^2 \end{cases}$$

$$\left(C_0 - C_1\right) | \left(K + C_0 - C_1\right) = \left(C_0 - C_1\right) | \left(C_1 - 3C_0 + C_0 - C_1\right) = \left(C_0 - C_1\right) | \left(-2C_0\right) = -2$$

$$-K| \left(C_0 - C_1\right) = \left(3C_0 - C_1\right) | \left(C_0 - C_1\right) = 3 - 1 = 2$$

$$D\frac{0}{6} = {}^{1}_{7}\mathrm{rat} \begin{cases} \alpha_1 = C_1 & \bcancel{1} \\ 3\alpha_0 + \alpha_1 = 3C_0 - 2C_1 & \bcancel{7} \end{cases} \begin{cases} \cancel{1} & D_0^+ \\ \overline{*\cancel{1}}_{-} & D_6^+ \end{cases} = \text{ ALF } \overset{1/g_s}{\underset{1/g_s}{\text{spin}}} \begin{cases} K_1/1 = D_0 = \dfrac{1}{g_s \ell_s} \\ K_6/0 = D_6 = \dfrac{1}{g_s \ell_s^7} \end{cases}$$

$$C_1| \left(K + C_1\right) = C_1|\overbrace{C_1 - 3C_0 + C_1} = C_1|\overbrace{2C_1 - 3C_0} = 2\,C_1|C_1 = -2$$

$$-K|C_1 = \overbrace{3C_0 - C_1}|C_1 = -C_1|C_1 = 1$$

$$\not{4}\frac{2}{4} = \not{4}\frac{2}{5} \boxminus \frac{0}{1} = {}^{3}_{5}\mathrm{rat} \begin{cases} \alpha_0 + \alpha_1 = C_0 & \bcancel{3} \sim \cancel{3} \\ 2\alpha_0 + \alpha_1 = 2C_0 - C_1 & \bcancel{5} \sim \overline{*\cancel{3}}_{-} \end{cases} \begin{cases} M_2/0 = D_2 = \dfrac{1}{g_s \ell_s^3} \sim 1/g_s \\ M_5/1 = D_4 = \dfrac{1}{g_s \ell_s^5} \sim 1/g_s \end{cases}$$

$$C_0| \left(K + C_0\right) = C_0| \left(C_1 - 3C_0 + C_0\right) = C_0| \left(C_1 - 2C_0\right) = -2$$

$$-K|C_0 = \left(3C_0 - C_1\right) |C_0 = 3$$

$$D\frac{-2}{8} = \frac{2C_1 - C_0}{4C_0 - 3C_1} \overset{1/g_s^0}{\underset{1/g_s^2}{\text{vect}}} \begin{cases} K_1/0 = K_1 = \dfrac{1}{R_i} \\ K_6/1 = K_5 = \dfrac{R_\tau^2}{g_s^2 \ell_s^8} \end{cases}$$

$$
{}_9\mathrm{rat} \left\{ 4\alpha_0 + \alpha_1 = 4C_0 - 3C_1 \right. \quad 9 \left\{ D_8^+ \sim \frac{R_\tau^2}{g_s^3 \ell_s^9} \right.
$$