

Ginsparg

$$\text{background} \quad \begin{cases} k \\ 2 \\ \mathbb{X}^\alpha \end{cases}$$

$$\text{Het world-sheet} \quad \begin{cases} \mathcal{X}^\mu & 1 \leqslant \mu \leqslant 8 \\ \mathcal{O}^\mu & 1 \leqslant \mu \leqslant 8 \\ \mathcal{Q}^\alpha & \alpha \in 16 \end{cases}$$

$$\boxed{\mathcal{X}^\mu \mathcal{O}^\mu \mathcal{Q}^\alpha}_{k^2 \mathbb{X}} = \mathcal{X}_\mu^\mu k_\nu k_* \mathcal{X}^\nu + \mathcal{X}_*^\mu \mathcal{O}^\nu_{\mu\nu} 2 + \mathcal{O}^\mu_\mu k_\nu k \mathcal{X}^\nu + \mathcal{Q}^\alpha_* \mathcal{Q}^\alpha + \mathcal{Q}^\alpha \mathcal{X}^\mu_\mu \mathbb{X}^\alpha \mathcal{Q}^\alpha$$

$$= \underbrace{\tau_{21} \mathcal{X}_1^\mu \mathcal{X}^\nu + \frac{1}{\tau_2} \mathcal{X}_2^\mu - \tau_{11} \mathcal{X}_1^\mu \mathcal{X}_2^\nu - \tau_{11} \mathcal{X}_1^\nu}_{\text{space-time lattice } \mathbb{R}^8 \supset \Lambda} \quad \begin{matrix} \mathbb{Z}^k \xrightarrow{\begin{matrix} \mathbf{l} \\ \vdots \\ \mathbf{l} \end{matrix}} \\ \cong n^i \mathbb{L}^i \end{matrix}$$

$$\mathcal{X}^{\mu + 2\pi} = \mathcal{X}^\mu + 2\pi i n^i \mathbb{L}^i$$

$$z = \sigma_1 + \tau \sigma_2$$

$$\frac{dz^* dz}{2i} = \frac{(d\sigma_1 + \frac{*}{\tau} d\sigma_2) \wedge (d\sigma_1 + \tau d\sigma_2)}{2i} = \frac{\frac{*}{\tau} d\sigma_2 \wedge d\sigma_1 + \tau d\sigma_1 \wedge d\sigma_2}{2i} = \frac{\tau - \frac{*}{\tau}}{2i} d\sigma_1 \wedge d\sigma_2 = \mathcal{I}(\tau) d\sigma_1 \wedge d\sigma_2$$