Ginsparg

Instanton sector

$$\flat = \flat^{i}\ {}_{i}\urcorner \in \Lambda \ni \natural = \natural^{i}\ {}_{i}\urcorner$$

$${}^{\sigma}\not{\theta}^{\mu} = \sigma_1 \flat^{\mu} + \sigma_2 \natural^{\mu}$$

$$\flat^{\mu}\ {}_{\mu}\not{1}^{I} = \flat^{I}$$

$$-\natural^{\mu}\ {}_{\mu}\not{1}^{I} = \natural^{I}$$

$$\flat\not{1}^{I} = \flat^{\mu}\ {}_{\mu}\not{1}^{I}$$

free fermion bdry cond

$$\frac{{}^{\sigma_1+1:\sigma_2}\not{Q}^{\alpha}}{{}^{\sigma_1:\sigma_2}\not{Q}^{\alpha}} = \exp\left(2\pi i \flat^{\mu}\ {}_{\mu}\not{1}^{\alpha}\right)$$

$$\frac{{}^{\sigma_1:\sigma_2+1}\not{Q}^{\alpha}}{{}^{\sigma_1:\sigma_2}\not{Q}^{\alpha}} = \exp\left(2\pi i \natural^{\mu}\ {}_{\mu}\not{1}^{\alpha}\right)$$

$$\det{}^{\llcorner}_{\natural} = {}^{\tau}\eta^{-1}\ {}^{\tau}\begin{bmatrix}\llcorner\\ \natural\end{bmatrix}$$

$$\tau \curvearrowright \tau + 1: \quad \begin{bmatrix}\llcorner\\ \natural\end{bmatrix} \curvearrowright \begin{bmatrix}\llcorner\\ \natural - \llcorner\end{bmatrix}$$

$$\tau \curvearrowright -\tau^{-1}: \quad \begin{bmatrix}\llcorner\\ \natural\end{bmatrix} \curvearrowright \begin{bmatrix}-\natural\\ \llcorner\end{bmatrix}$$

$${}^{\tau}\begin{bmatrix}\flat^{\mu}\ {}_{\mu}\not{1}^{I}\\ -{}_{5}\natural^{\mu}\ {}_{\mu}j^{I}\end{bmatrix}_{\alpha\beta}$$

$$\sum_{\alpha\beta}^{2\times 2}\prod_{I}^{16}\ {}^{\tau}\begin{bmatrix}\flat^{\mu}\ {}_{\mu}\not{1}^{I}\\ -\natural^{\mu}\ {}_{\mu}\not{1}^{I}\end{bmatrix}_{\alpha\beta}$$

$$\prod_{I}^{16}\ {}^{\tau}\begin{bmatrix}\flat^{\mu}\ {}_{\mu}\not{1}^{I}\\ -\natural^{\mu}\ {}_{\mu}\not{1}^{I}\end{bmatrix}_{00} + \prod_{I}^{16}\ {}^{\tau}\begin{bmatrix}\flat^{\mu}\ {}_{\mu}\not{1}^{I}\\ -\natural^{\mu}\ {}_{\mu}\not{1}^{I}\end{bmatrix}_{01} + \prod_{I}^{16}\ {}^{\tau}\begin{bmatrix}\flat^{\mu}\ {}_{\mu}\not{1}^{I}\\ -\natural^{\mu}\ {}_{\mu}\not{1}^{I}\end{bmatrix}_{10} + \prod_{I}^{16}\ {}^{\tau}\begin{bmatrix}\flat^{\mu}\ {}_{\mu}\not{1}^{I}\\ -\natural^{\mu}\ {}_{\mu}\not{1}^{I}\end{bmatrix}_{11}$$

$$\text{Fourier } \mathcal{E}_{\square} \overset{\mathbb{R}^8}{|} \square = \int_{d\square}^{\mathbb{R}^8} \mathfrak{e}^{2\pi i \square \square \square} \square$$

$$\text{Poisson } \sum_{\Lambda}^{\square} \square\square = \sum_{\Lambda^+}^{\square} \mathcal{E}_{\square} \overset{\mathbb{R}^8}{|} \square$$

$$\square\square = {}^{-2\pi}\left(\frac{\overbrace{\square - \tau_1 \square}^{2}}{\tau_2} + i\square \left(2\square\square + \left(m^I + \frac{\square\square^I}{2} \right) \square^I \right) \right)$$

$$\Longrightarrow \mathcal{E}_{\square} \overset{\mathbb{R}^8}{|} \square = {}^{-\pi}\left(\frac{\tau_2}{2} \overbrace{2\square\square + \left(m^I + \frac{\square\square^I}{2} \right) \square^I - \square}^{2} + 2i\tau_1 \square \left(2\square\square + \left(m^I + \frac{\square\square^I}{2} \right) \square^I - \square \right) \right)$$

$$\sum^{\square\square} {}^{-2\pi}\left(\tau_2 \square^2 + \frac{\overbrace{\square - \tau_1 \square}^{2}}{\tau_2} + 2i\square\square\square \right) \det$$

$$= \sum^{\square\square} {}^{-2\pi}\left(\tau_2 \square^2 + \frac{\overbrace{\square - \tau_1 \square}^{2}}{\tau_2} + 2i\square\square\square \right) \sum_{m^I}^{\mathbb{Z}_+^{8k} \cup \overline{\mathbb{Z}+1/2}_+^{8k}} {}^{\pi i}\left(\tau \overbrace{m^I + \square\square^I}^{2} - 2 \left(m^I + \frac{\square\square^I}{2} \right) \square\square^I \right)$$

$$= \sum^{\square} \exp -2\pi \tau_2 \square^2 \sum_{m^I}^{\mathbb{Z}_+^{8k} \cup \overline{\mathbb{Z}+1/2}_+^{8k}} {}^{\tau\pi i} \overbrace{m^I + \square\square^I}^{2} \sum^{\square} {}^{-2\pi}\left(\frac{\overbrace{\square - \tau_1 \square}^{2}}{\tau_2} + i\square \left(2\square\square + \left(m^I + \frac{\square\square^I}{2} \right) \square^I \right) \right)$$

$$\sum^{\square} \exp -2\pi \tau_2 \square^2 \sum^{\square} {}^{-2\pi}\left(\frac{\overbrace{\square - \tau_1 \square}^{2}}{\tau_2} + i\square \left(2\square\square + \left(m^I + \frac{\square\square^I}{2} \right) \square^I \right) \right)$$

$$= \sum^{\square} \exp -2\pi \tau_2 \square^2 \sum_{\Lambda^+}^{\square} {}^{-\pi}\left(\frac{\tau_2}{2} \overbrace{2\square\square + \left(m^I + \frac{\square\square^I}{2} \right) \square^I - \square}^{2} + 2i\tau_1 \square \left(2\square\square + \left(m^I + \frac{\square\square^I}{2} \right) \square^I - \square \right) \right)$$

$$\sum {}^{\pi i\left(\tau_1 + i\tau_2\right)} \overbrace{\frac{\text{ꓶ} - 2\flat\cancel{2} - \left(m^I + \frac{\flat\cancel{1}^I}{2}\right)\cancel{1}^I}{2} + \flat}^{2} {}^{-\pi i\left(\tau_1 - i\tau_2\right)} \overbrace{\frac{\text{ꓶ} - 2\flat\cancel{2} - \left(m^I + \frac{\flat\cancel{1}^I}{2}\right)\cancel{1}^I}{2} - \flat}^{2}$$

$$= \sum {}^{\pi i\tau_1} \left(\overbrace{\frac{\text{ꓶ} - 2\flat\cancel{2} - \left(m^I + \frac{\flat\cancel{1}^I}{2}\right)\cancel{1}^I}{2} + \flat}^{2} - \overbrace{\frac{\text{ꓶ} - 2\flat\cancel{2} - \left(m^I + \frac{\flat\cancel{1}^I}{2}\right)\cancel{1}^I}{2} - \flat}^{2} \right)$$

$${}^{-\pi\tau_2} \left(\overbrace{\frac{\text{ꓶ} - 2\flat\cancel{2} - \left(m^I + \frac{\flat\cancel{1}^I}{2}\right)\cancel{1}^I}{2} + \flat}^{2} + \overbrace{\frac{\text{ꓶ} - 2\flat\cancel{2} - \left(m^I + \frac{\flat\cancel{1}^I}{2}\right)\cancel{1}^I}{2} - \flat}^{2} \right)$$

$$= \sum {}^{2\pi i\tau_1} \left(\flat \left(\text{ꓶ} - 2\flat\cancel{2} - \left(m^I + \frac{\flat\cancel{1}^I}{2}\right)\cancel{1}^I \right) \right) {}^{-\pi\tau_2} \left(\frac{1}{2} \overbrace{\text{ꓶ} - 2\flat\cancel{2} - \left(m^I + \frac{\flat\cancel{1}^I}{2}\right)\cancel{1}^I}^{2} + 2\flat^2 \right)$$