

$$\begin{aligned} {}^zU_w &= {}^zB_w^{-1/2} \mathfrak{t}_{-w}^* \mathfrak{t}_{-z} {}^zB_w^{-1/2} = \frac{z\overset{*}{w}}{-\overset{*}{w}} \left| \begin{array}{c|c} -z & \\ \hline \underbrace{1 + \overset{*}{w}\underline{z}}_{-1} \overset{*}{w}\underline{z} & -1 \end{array} \right\} \in G^{\mathbb{C}} \\ {}^zU_w^{-1} &= \frac{\overset{-1}{z\overset{*}{w}} \overset{-1}{1 + z\overset{*}{w}}}{\overset{*}{w}} \left| \begin{array}{c|c} z & \\ \hline \overset{*}{w} & \overset{*}{w}z \end{array} \right. \end{aligned}$$

$$\begin{aligned} \cancel{\mathcal{Z}} \stackrel{\text{LQO}}{=} \mathfrak{t}_z^* {}^zB_z^{1/2} \mathfrak{t}_z &= \frac{1}{\overset{*}{z}} \left| \begin{array}{c|c} 0 & \overset{-1/2}{z\overset{*}{z}} \\ \hline \overset{*}{z}\overset{*}{z} & 0 \end{array} \right| \frac{0}{\overset{*}{z}\overset{*}{z}} \left| \begin{array}{c|c} 1 & z \\ \hline 0 & 1 \end{array} \right. = \frac{\overset{-1/2}{z\overset{*}{z}}}{\overset{*}{z}\overset{*}{z}} \left| \begin{array}{c|c} \overset{-1/2}{z\overset{*}{z}} & z \\ \hline \overset{*}{z}\overset{*}{z} & -1/2 \end{array} \right. \\ {}^0U_{\cancel{\mathcal{Z}}} &= \mathfrak{t}_0 \cancel{\mathcal{Z}} \mathfrak{t}_z^{-1} = \mathfrak{t}_z^* {}^zB_z^{1/2} = \frac{1}{\overset{*}{z}} \left| \begin{array}{c|c} 0 & \overset{-1/2}{z\overset{*}{z}} \\ \hline \overset{*}{z}\overset{*}{z} & 0 \end{array} \right| \frac{0}{\overset{*}{z}\overset{*}{z}} \left| \begin{array}{c|c} 0 & \\ \hline \overset{*}{z}\overset{*}{z} & 1/2 \end{array} \right. = \frac{\overset{-1/2}{z\overset{*}{z}}}{\overset{*}{z}\overset{*}{z}} \left| \begin{array}{c|c} 0 & \\ \hline \overset{*}{z}\overset{*}{z} & 1/2 \end{array} \right. \\ {}^0\overline{U}_{\cancel{\mathcal{Z}}}^{-1} &= {}^zB_z^{-1/2} \mathfrak{t}_{-z} = \frac{\overset{1/2}{z\overset{*}{z}}}{0} \left| \begin{array}{c|c} 0 & 1 \\ \hline \overset{*}{z}\overset{*}{z} & -\overset{*}{z} \end{array} \right| \frac{0}{1} = \frac{\overset{1/2}{z\overset{*}{z}}}{-\overset{*}{z}\overset{*}{z}} \left| \begin{array}{c|c} 0 & \\ \hline \overset{*}{z}\overset{*}{z} & -1/2 \end{array} \right. \\ {}^zU_z &= {}^0\overline{U}_{\cancel{\mathcal{Z}}}^{-1} {}^0\overline{U}_{\cancel{\mathcal{Z}}}^* = {}^zB_z^{-1/2} \mathfrak{t}_{-z}^* {}^zB_z^{-1/2} = \frac{\overset{1/2}{z\overset{*}{z}}}{-\overset{*}{z}\overset{*}{z}} \left| \begin{array}{c|c} 0 & \overset{1/2}{z\overset{*}{z}} \\ \hline \overset{*}{z}\overset{*}{z} & 0 \end{array} \right| \frac{-\overset{-1/2}{z\overset{*}{z}} z}{\overset{*}{z}\overset{*}{z}} = \frac{z\overset{*}{z}}{-\overset{*}{z}} \left| \begin{array}{c|c} -z & \\ \hline \overset{*}{z}\overset{*}{z} & -1 \end{array} \right. \end{aligned}$$

$$\zeta {}^zU_w = \widehat{z + \zeta}^w - 2z^w$$

$$\begin{aligned} \zeta {}^zU_w + 2z^w &= \underbrace{\zeta \overset{-1}{z + \zeta} \overset{*}{w}}_{-\zeta} \underbrace{\zeta \cancel{1 + \overset{*}{w}z} \cancel{\overset{*}{w}z} - z}_{-1} + 2 \cancel{\overset{*}{z}\overset{*}{w}} z = \underbrace{\zeta \overset{-1}{z + \zeta} \overset{*}{w}}_{-\zeta} \underbrace{\zeta \cancel{1 + \overset{*}{w}z} - z \cancel{\overset{*}{w}z}}_{-1} \cancel{\overset{*}{w}z} + 2z \cancel{\overset{*}{w}z}_{-1} \\ &= \underbrace{\zeta \overset{-1}{z + \zeta} \overset{*}{w}}_{-\zeta} \underbrace{\zeta \cancel{1 + \overset{*}{w}z} - z \cancel{\overset{*}{w}z} + 2 \cancel{z + \zeta} \overset{*}{w} z}_{-1} \cancel{\overset{*}{w}z} = \underbrace{\zeta \overset{-1}{z + \zeta} \overset{*}{w}}_{-\zeta} \underbrace{\zeta + z - \cancel{z + \zeta} \overset{*}{w}}_{-1} \cancel{\overset{*}{w}z} = \underbrace{\zeta \overset{-1}{z + \zeta} \overset{*}{w}}_{-\zeta} \underbrace{\zeta + z}_{-1} = \widehat{z + \zeta}^w \end{aligned}$$

$${}^z U_w^n B_{-\omega}^n = B_{-\dot{\omega}}^n \overbrace{z \underline{w + \omega}}^* {}^n$$

$$\dot{\omega} = \overbrace{\underline{w + \omega} z}^{-1} \omega \overbrace{\underline{z w}}^{-1} - w^z$$

$$\begin{aligned} {}^\zeta \text{LHS} &= \underbrace{\zeta \underline{z + \zeta \dot{w}}}^n \underbrace{1 + \zeta \underline{z + \zeta \dot{w}}}^{-1} \underbrace{\zeta \underline{1 + \dot{w}z \dot{w}z}}^{-1} - z \dot{\omega} = \underbrace{\zeta \underline{z + \zeta \dot{w}}}^n \underbrace{\zeta \underline{1 + \dot{w}z \dot{w}z}}^{-1} - z \dot{\omega} \\ &= z \dot{w} - z \dot{\omega} + \zeta \underbrace{\dot{w}z \dot{w}z \dot{\omega} - \dot{w}}_n = 1 + \zeta \underbrace{\dot{w}z \dot{w}z \dot{\omega} - \dot{w}}^{-1} \underbrace{z \dot{w} - z \dot{\omega}}_n \\ \dot{\omega} &= \overbrace{\dot{w}z \dot{w}z \dot{\omega} - \dot{w}}^* \underbrace{z \dot{w} - z \dot{\omega}}_{-1} = \underbrace{w \dot{z} - \omega \dot{z}}_{-1} \underbrace{\omega \dot{z} w \underline{1 + \dot{z}w} - w}_{-1} \\ &= \underbrace{\underline{w + \omega} z}^{-1} \underbrace{\omega - w + \omega \underline{w z}}^{-1} \underbrace{\dot{z} w}_z = \underbrace{(w + \omega) \dot{z} \omega - w}_{-1} \underbrace{\dot{z} w}_z = \underbrace{(w + \omega) \dot{z} \omega \dot{z} w}_z - w^z \end{aligned}$$

$$\begin{aligned} {}^z U_w^{-1} B_{-\omega}^n &= \overbrace{\zeta \dot{w} \underline{1 + z \dot{w}} + z \dot{\omega}}_{-1} {}^n B_{-\dot{\omega}} \\ \dot{\omega} &= \overbrace{\omega \dot{z} + \underline{1 + w z * w \dot{z}}}^{-1} \overbrace{w + \omega \underline{z \dot{w}}} \end{aligned}$$