$$\mathbb{S}^2 = O_{+2}/O_2 = \frac{x = x^0{:}x^1{:}x^2}{{}^2x^0 + {}^2x^1 + {}^2x^2 = 1}$$

$$L^2\left(\mathbb{S}^2\right) = \sum_{m}^{\mathbb{N}} L^2_m\left(\mathbb{S}^2\right)$$

$$L^2_m\left(\mathbb{S}^2\right) = \mathbb{C}\frac{\overbrace{a|x}^{m}}{a \in \mathbb{C}^{+2}: \quad a|\bar{a} = 0} = \left(O_{+2}\right)_m$$

$$\overbrace{a|x}^{m}: \quad a \in \mathbb{C}^{+2}$$

$$\partial_i \overbrace{a|x}^{m} = m \overbrace{a|x}^{m-} a|e_i$$

$$\partial_i^2 \overbrace{a|x}^{m} = m(m-) \overbrace{a|x}^{m-2} \underbrace{a|e_i}\underbrace{a|e_i} = m(m-) \overbrace{a|x}^{m-2} \underbrace{a|e_i}\underbrace{e_i|\bar{a}}$$

$$\sum_{i}^{+2} \partial_i^2 \overbrace{a|x}^{m} = m(m-) \overbrace{a|x}^{m-2} \sum_{i}^{+2} \underbrace{a|e_i}\underbrace{e_i|\bar{a}} = m(m-) \overbrace{a|x}^{m-2} \underbrace{a|\bar{a}}$$

$$\Omega_m(x) = \overbrace{x^0 + ix^1}^{m}$$

$$x^0 = \frac{z + \bar{z}}{\bar{z}z + 1}$$

$$x^1 i = \frac{z - \bar{z}}{\bar{z}z + 1}$$

$$x^2 = \frac{\bar{z}z - 1}{\bar{z}z + 1}$$

$$\Omega_m(z) = \left(\frac{2z}{\bar{z}z + 1}\right)^m$$

$$\partial \Omega_m = \frac{mz^{m-}}{\left(1 + \bar{z}z\right)^{m+}}$$

$$\bar{\partial}\,\Omega_m = -\frac{m z^{m+}}{\left(1+\bar{z}z\right)^{m+}}$$

$$z^2\,\partial\,\Omega_m + \bar{\partial}\,\Omega_m = 0$$

$$z\partial\,\Omega_m - \bar{z}\,\bar{\partial}\,\Omega_m = m\,\Omega_m$$