$$\text{linear Recursion } F_0 = 0 / F_1 = 1 / n \geqslant 2: \quad F_n = F_{n-1} + F_{n-2}$$

$$F(x) = \sum_{n \geqslant 0} F_n x^n = x + \sum_{n \geqslant 2} F_n x^n$$

$$F(x) = \frac{x}{1 - x - x^2}$$

$$F(x) - x = \sum_{n \geqslant 2} F_n x^n = \sum_{n \geqslant 2} \left(F_{n-1} + F_{n-2}\right) x^n = \sum_{n \geqslant 2} F_{n-1} x^n + \sum_{n \geqslant 2} F_{n-2} x^n$$

$$= x \sum_{n \geqslant 2} F_{n-1} x^{n-1} + x^2 \sum_{n \geqslant 2} F_{n-2} x^{n-2} = x \sum_{m \geqslant 1} F_m x^m + x^2 \sum_{m \geqslant 0} F_m x^m$$

$$= xF(x) + x^2 F(x) = \left(x + x^2\right) F(x) \Longrightarrow F(x) \left(1 - x - x^2\right) = x$$

$$\text{Factorization } 1 - x - x^2 = (1 - ax)(1 - bx)$$

$$a = \frac{1 + \sqrt{5}}{2} \text{ golden ratio :} \quad b = \frac{1 - \sqrt{5}}{2}$$

$$1 - x - x^2 = (1 - ax)(1 - bx) = 1 - (a + b)x + abx^2 \Longrightarrow a + b = 1 \Longrightarrow b = 1 - a$$

$$ab = a(1 - a) = -1 \Longrightarrow a^2 - a - 1 = 0$$

$$a = \frac{1}{2} + \sqrt{\frac{1}{4} + 1} = \frac{1 + \sqrt{5}}{2}: \quad b = 1 - a = \frac{1 - \sqrt{5}}{2}$$

$$\text{part fractions } F(x) = \frac{x}{1 - x - x^2} = \frac{1/\sqrt{5}}{1 - ax} - \frac{1/\sqrt{5}}{1 - bx}$$

$$F(x) = \frac{c}{1 - ax} - \frac{d}{1 - bx} = \frac{c(1 - bx) - d(1 - ax)}{(1 - ax)(1 - bx)} = \frac{(c - d) + (ad - bc)x}{1 - x - x^2}$$

$$\Longrightarrow (c - d) + (ad - bc)x = x \Longrightarrow c = d: \quad 1 = ad - bc = d(a - b) = c\sqrt{5} \Longrightarrow c = 1/\sqrt{5}$$

$$F_n = \frac{a^n - b^n}{\sqrt{5}} = \frac{1}{\sqrt{5}} \left(\left(\frac{1+\sqrt{5}}{2} \right)^n - \left(\frac{1-\sqrt{5}}{2} \right)^n \right)$$

closed form/no summation/why integer / all $n \geqslant 0$/ why recursion

$$F(x) = \frac{1}{\sqrt{5}} \sum_{n \geqslant 0} (ax)^n - \frac{1}{\sqrt{5}} \sum_{n \geqslant 0} (bx)^n = \frac{1}{\sqrt{5}} \sum_{n \geqslant 0} \left(a^n - b^n \right) x^n$$

$$\mathbb{N} \supset M \text{ azush } \Leftrightarrow i \in M \Longrightarrow i+ \notin M$$

$$\mathcal{F}_M = \frac{A \subset M}{A \text{ azush}}$$

$$\sharp \mathcal{F}_{0|n+} = \sharp \mathcal{F}_{0|n} + \sharp \mathcal{F}_{0|n-}$$

$$\text{azush } A \in 0|n+$$

$$\text{if } n+ \notin A \Longrightarrow 0|n \supset A \text{ azush } \Longrightarrow \sharp \mathcal{F}_{0|n}$$

$$\text{if } n+ \in A \Longrightarrow n \notin A \Longrightarrow 0|n- \supset A \llcorner n+ \text{ azush } \Longrightarrow \sharp \mathcal{F}_{0|n-}$$