$$M \subset \mathbb{N} \supset N \text{ zush } = \text{ number intervals}$$

$$M = 1|r \text{ number interval}$$

$$M^N = \{N \xrightarrow{\sim} M\}$$

$$\mathbb{N}^{1|r} = \frac{\underline{k} = k_1 \cdots k_r \text{ r-tuples}}{k_i \geqslant 0}$$

$$\text{multi-subset}: \quad x_1^{k_1} \cdots x_n^{k_n}$$

$$\begin{array}{ccc} \underline{k} \in & \mathbb{N}^{1|r} & \ni \Delta \underline{m} \\ \text{integration} \downarrow & & \uparrow \text{differentiation} \\ \Sigma \underline{k} \in & \mathbb{N}^{1|r}_{\geqslant} & \ni \underline{m} \end{array}$$

$$k_1 \cdot\cdot k_r \geqslant 0$$

$$m_i = k_i + \cdot\cdot + k_r \text{ integration}$$

$$m_1 \geqslant m_2 \geqslant \cdot\cdot \geqslant m_r \geqslant 0 = m_{r+}$$

$$k_i = m_i - m_{i+} \text{ differentiation}$$

$$\mathbb{N}^{0|r}_k = \frac{k_0 \cdots k_r \geqslant 0}{k_0 + \cdot\cdot + k_r = k} \text{ restricted tuples}$$

$$\underline{k} \in \qquad \mathbb{N}_k^{0|r} \qquad \ni \Delta\underline{m}$$

integration $\downarrow$ $\uparrow$ differentiation

$$\Sigma\underline{k} \in \qquad (0|k)^{1|r}_{\geqslant} \qquad \ni \underline{m}$$

$$k_0 \cdot\cdot k_r \geqslant 0: \quad k_0 + \cdot\cdot + k_r = k$$

$$1 \leqslant i \leqslant r: \quad m_i = k_i + \cdot\cdot + k_r$$

$$\Longrightarrow m_1 = k_1 + \cdot\cdot + k_r \leqslant k_0 + k_1 + \cdot\cdot + k_r = k$$

$$k \geqslant m_1 \geqslant m_2 \geqslant \cdot\cdot \geqslant m_r \geqslant 0 = m_{r+}$$

$$1 \leqslant i \leqslant r: \quad k_i = m_i - m_{i+}: \quad k_0 = k - m_1 \geqslant 0$$

$$k_0 + k_1 + \cdot\cdot k_r = \big(k - m_1\big) + \big(m_1 - m_2\big) + \cdot\cdot + \Big(m_{r-} - m_r\Big) + m_r = k$$

subset $0 \leqslant k_i \leqslant 1$

$$\sharp\frac{k_0 \cdots k_r \geqslant 0}{k_0 + \cdot\cdot + k_r = k} = \begin{bmatrix} k+r \\ r \end{bmatrix} = \begin{bmatrix} k+r \\ k \end{bmatrix}$$

bijection : $\quad k_0 \,()\, k_1 \,()\, \cdot\cdot \,()\, k_{r-} \,()\, k_r: \quad r$ gaps in $k+r$ set