$$z{:}v \in T(D): \quad v \in T_z(D)$$

$$\underline{P}_z v \text{ herm}$$

$$\overbrace{\nabla_v \Phi}_z = P_z \overbrace{\underline{\Phi}_z v} = \underline{\Phi}_z v - \overbrace{\underline{P}_z v}\, \Phi_z \in H_z$$

$$\underline{\Phi}_z v = \underline{P_z \Phi_z} v = \overbrace{\underline{P}_z v}\, \Phi_z + P_z \overbrace{\underline{\Phi}_z v}$$

$$\overbrace{\nabla_v \Phi}_0 = P \overbrace{\underline{\Phi}_0 v} = \underline{\Phi}_0 v - \overbrace{\underline{P}_0 v}\, \Phi_0 \in H_0$$

$$D \xrightarrow[\text{section}]{\nabla_X \Phi} L^2\left(S_\ell^{\mathbb{C}}\right)$$

$$\overbrace{\nabla_X \Phi}_z = \nabla_{X_z} \Phi_z = P_z \overbrace{\underline{\Phi}_z X_z}$$

$$\nabla_{g_* X}(g \cdot \Phi) = g \cdot \left(\nabla_X \Phi\right)$$

$$\underline{g \cdot \Phi}_{g \cdot z}\, \underline{g}_z v = \hat{U}_g\, \underline{\Phi}_z v$$

$$\Longrightarrow \nabla_{\underline{g}_z v}(g \cdot \Phi)_{g \cdot z} = P_{g \cdot z}\, \underline{g \cdot \Phi}_{g \cdot z}\, \underline{g}_z v = P_{g \cdot z}\, \hat{U}_g\, \underline{\Phi}_z v = \hat{U}_g\, P_z\, \underline{\Phi}_z v = \hat{U}_g\, (\nabla_v \Phi)_z$$

$$\Longrightarrow \overbrace{\nabla_{g_* X} \underbrace{g \cdot \Phi}}_{g \cdot z} = \overbrace{\nabla_{\overbrace{g_* X}_{g \cdot z}} \underbrace{g \cdot \Phi}}_{g \cdot z} = \overbrace{\nabla_{\underline{g}_z X_z} \underbrace{g \cdot \Phi}}_{g \cdot z} = \hat{U}_g\, \overbrace{\nabla_{X_z} \Phi}_z = \hat{U}_g\, \overbrace{\nabla_X \Phi}_z = \underbrace{g \cdot \overbrace{\nabla_X \Phi}}_{g \cdot z}$$